Communication Separations for Truthful Auctions:

Breaking the Two-Player Barrier

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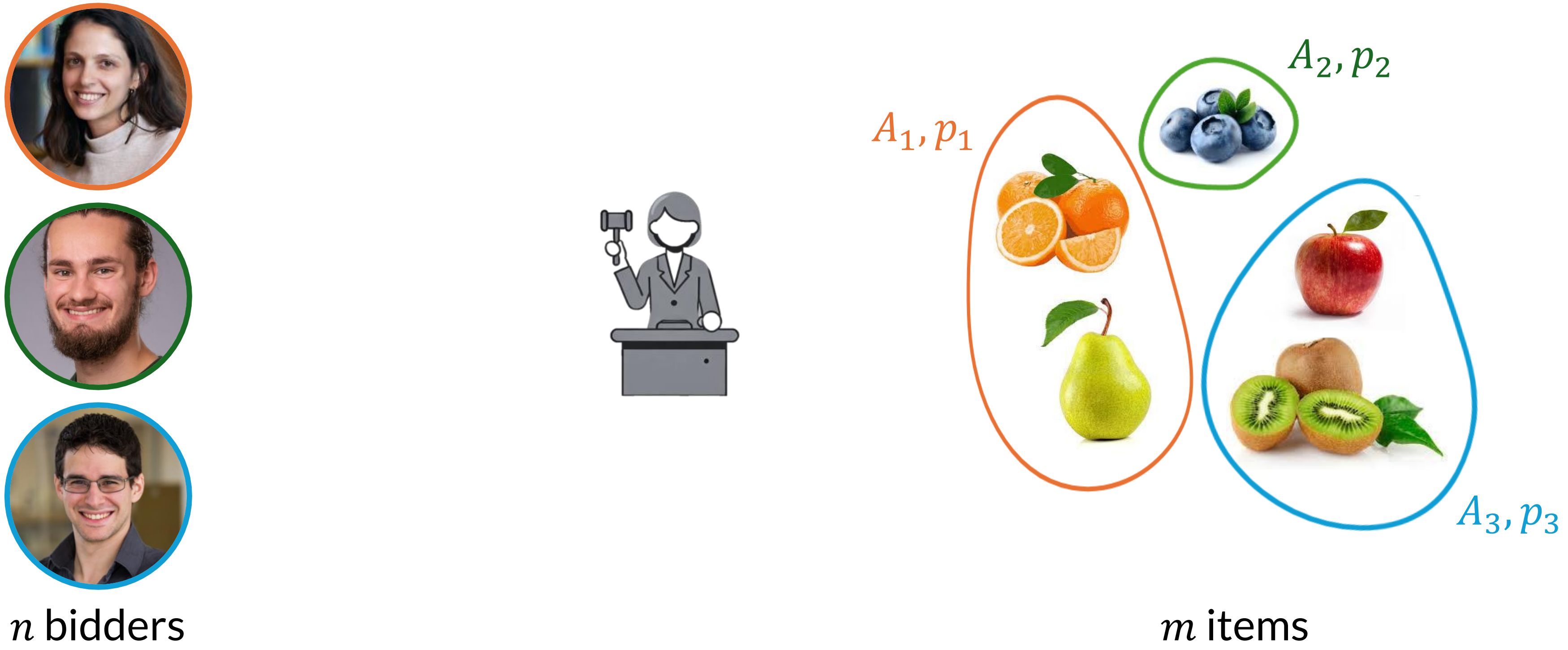
Problem setup: combinatorial auction

n bidders

m items

Problem setup: combinatorial auction



- Auctioneer want to allocate items $A_i \subseteq [m]$ to bidder $i \in [n]$ and charges price p_i

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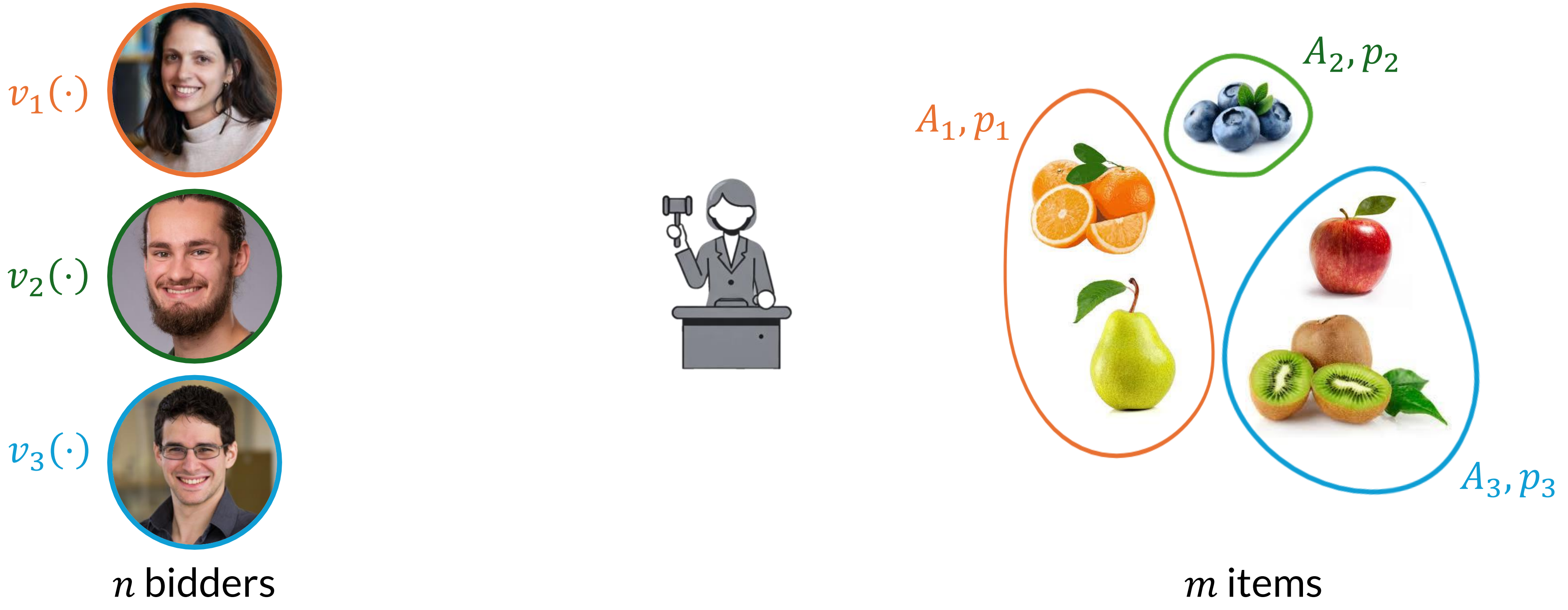

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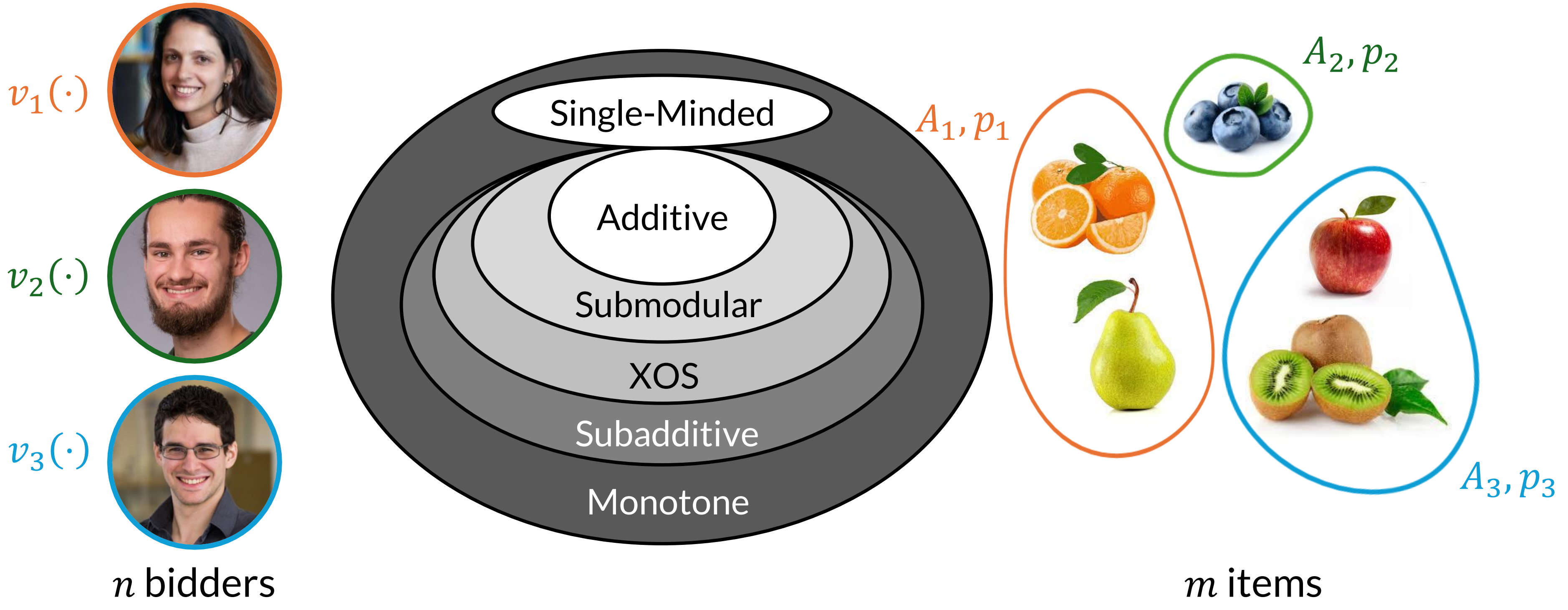

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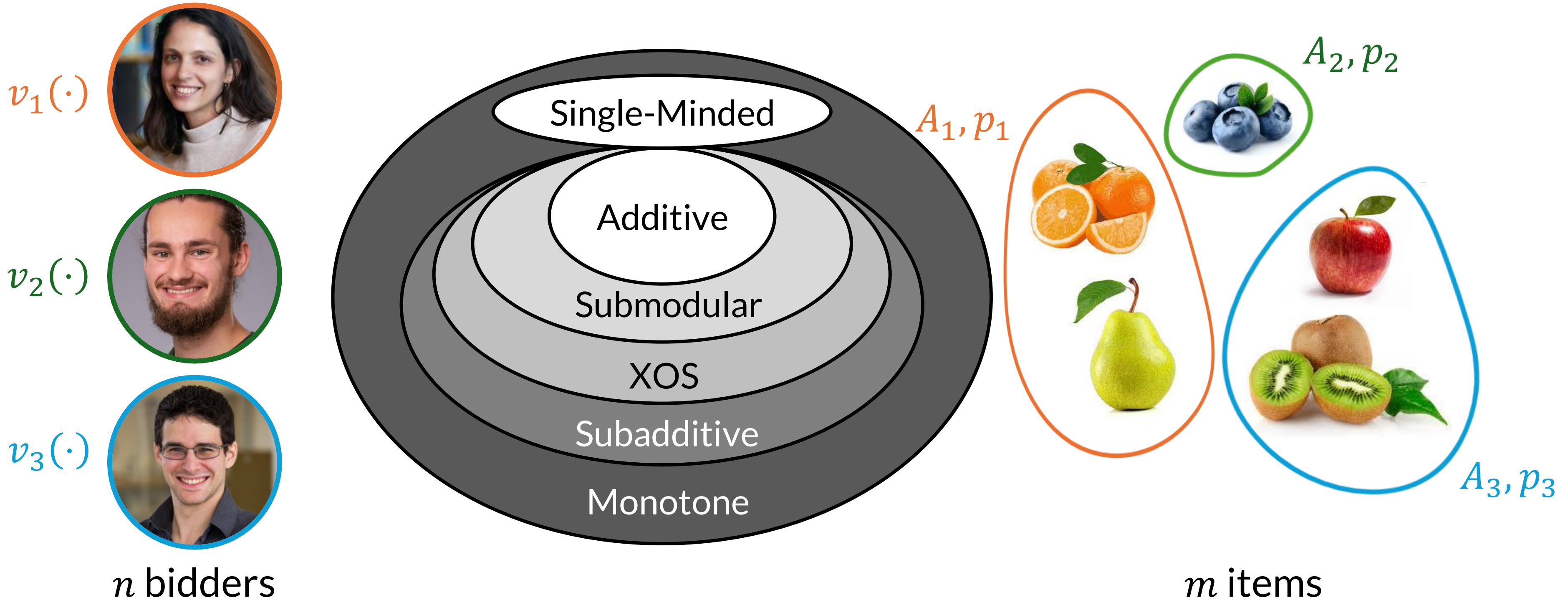

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  - Might take $\exp(m)$ bits to describe

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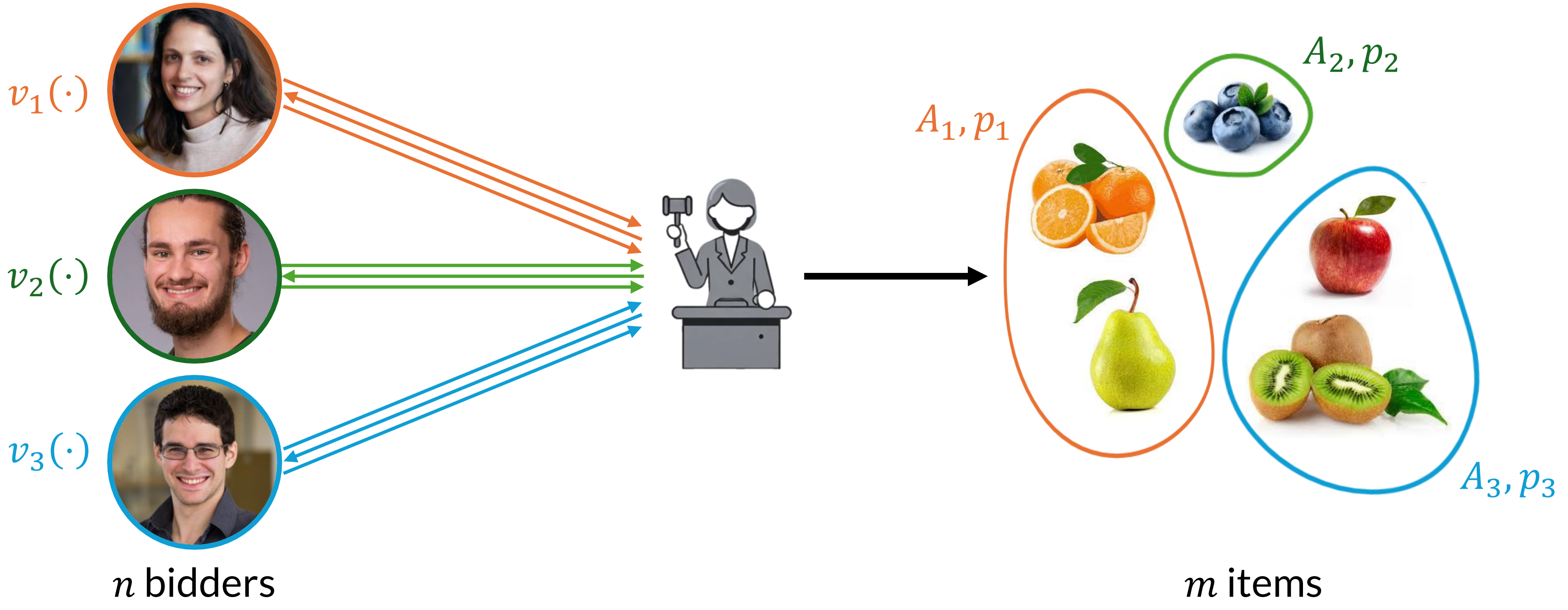

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- Each bidder i has **private** valuation function $v_i: 2^{[m]} \to \mathbb{R}_{\geq 0}$ **in some class $\mathcal{V}$**
- Bidders participate in some **interactive protocol**

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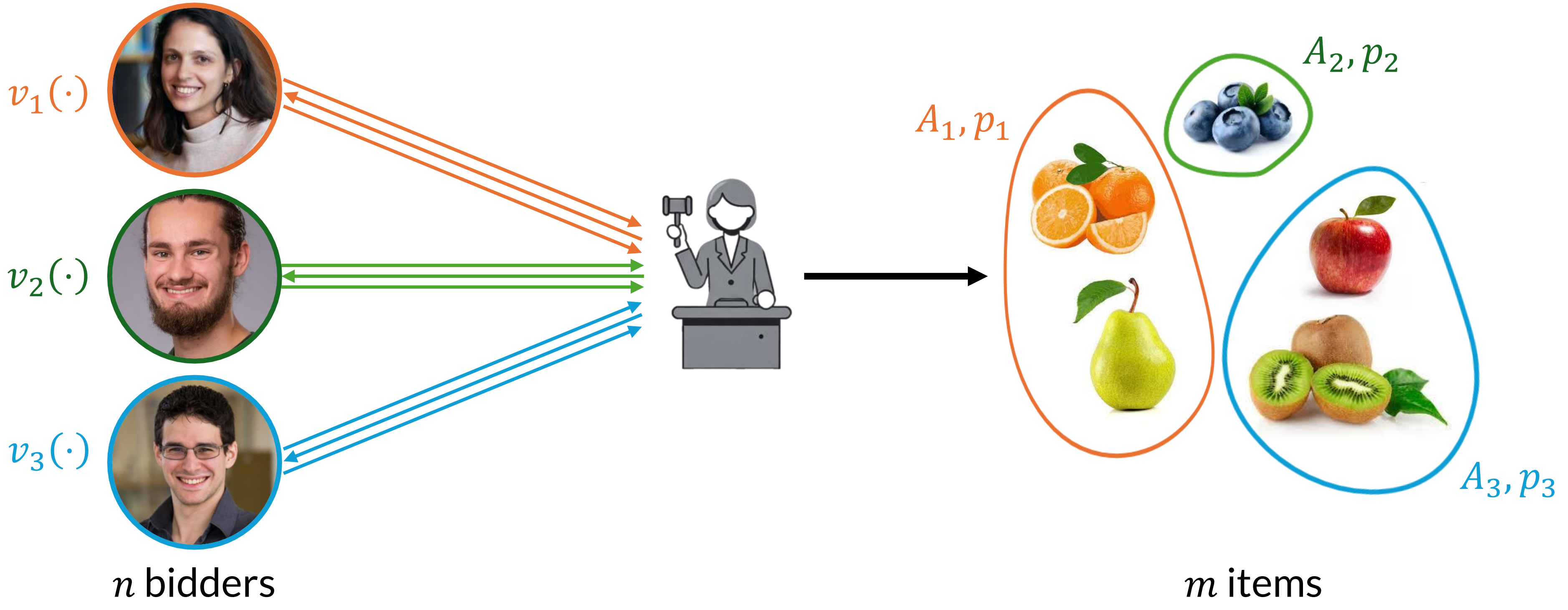

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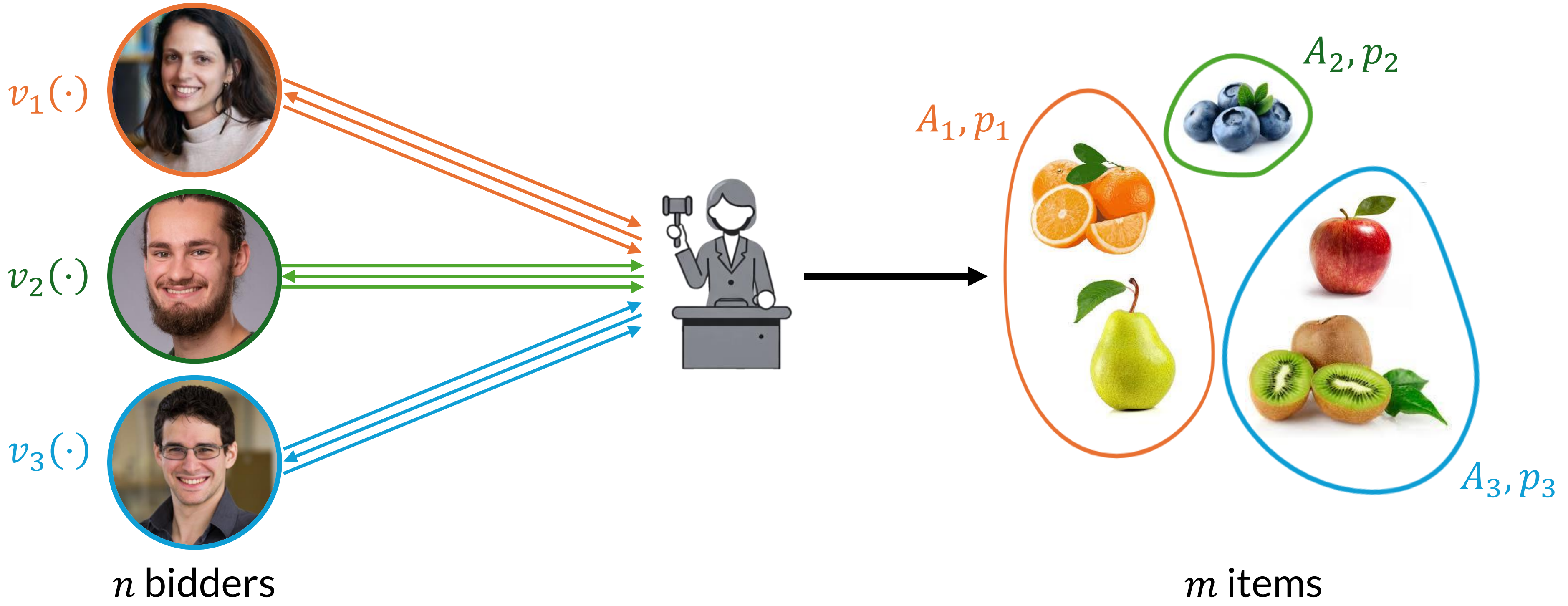

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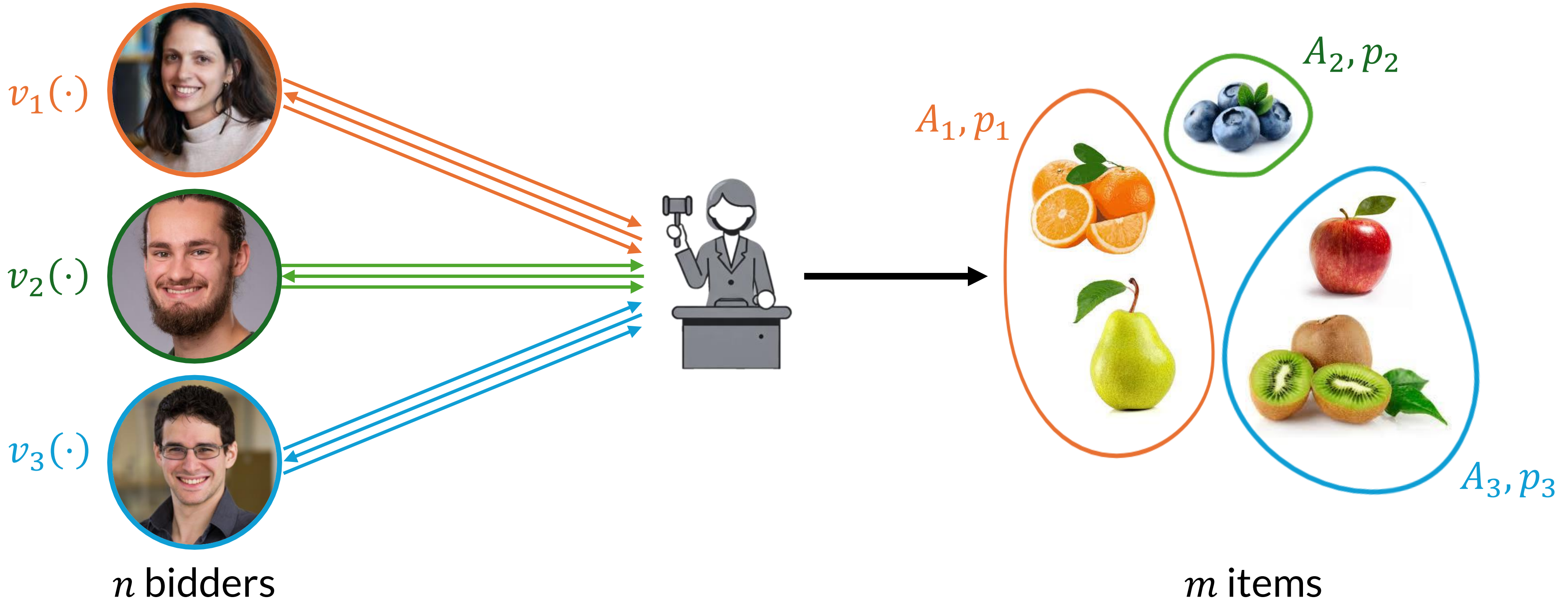

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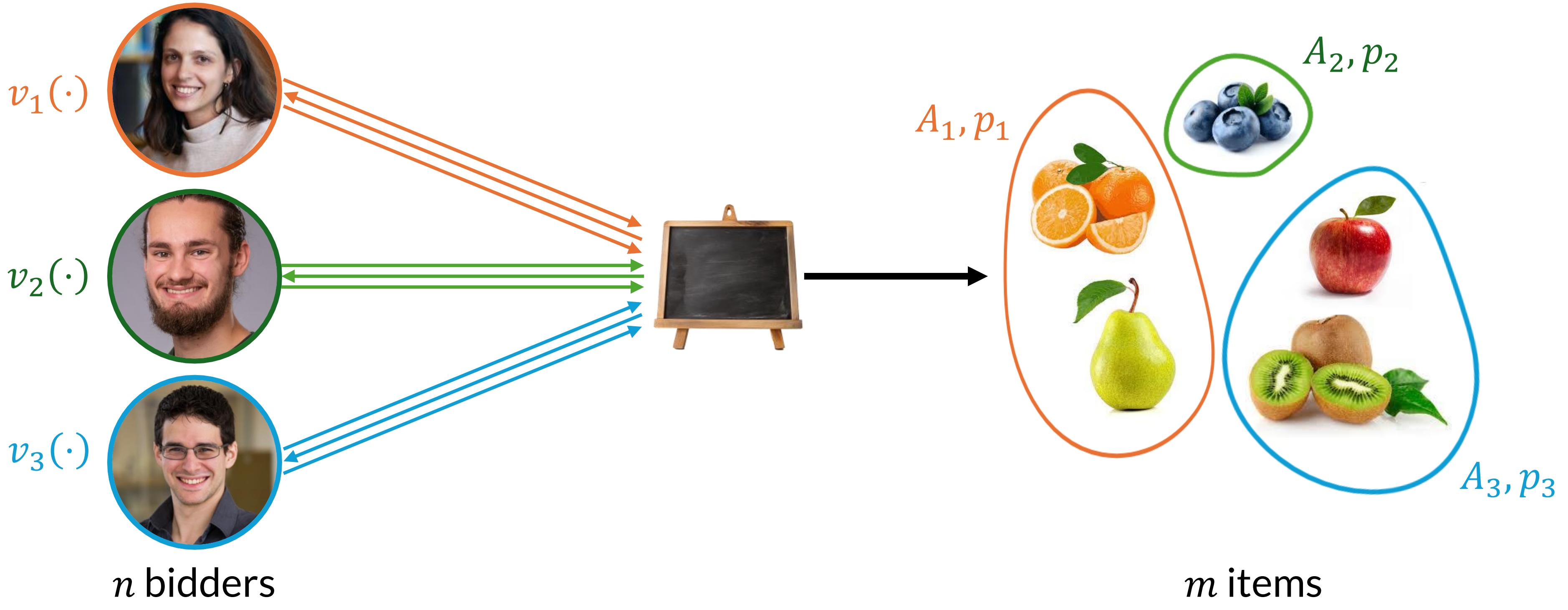
Bidders incentivized to follow the protocol

Problem setup: communication model



- n-player deterministic blackboard communication

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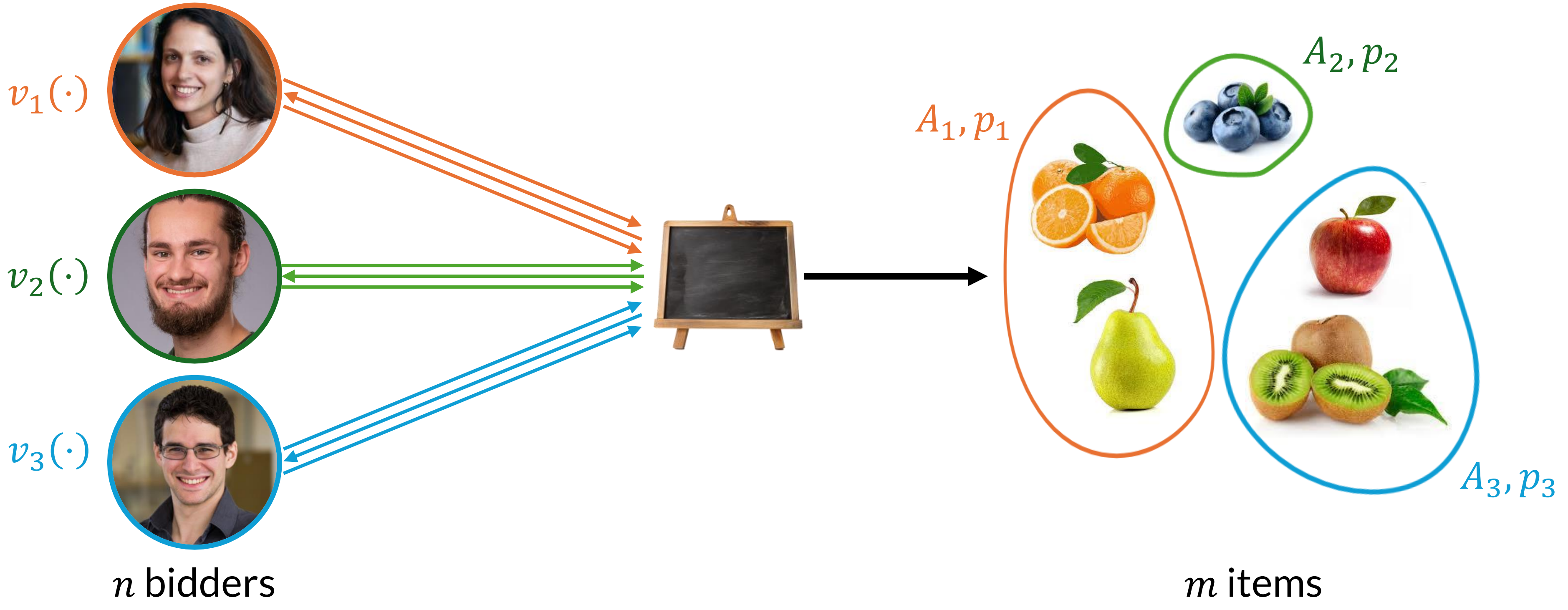
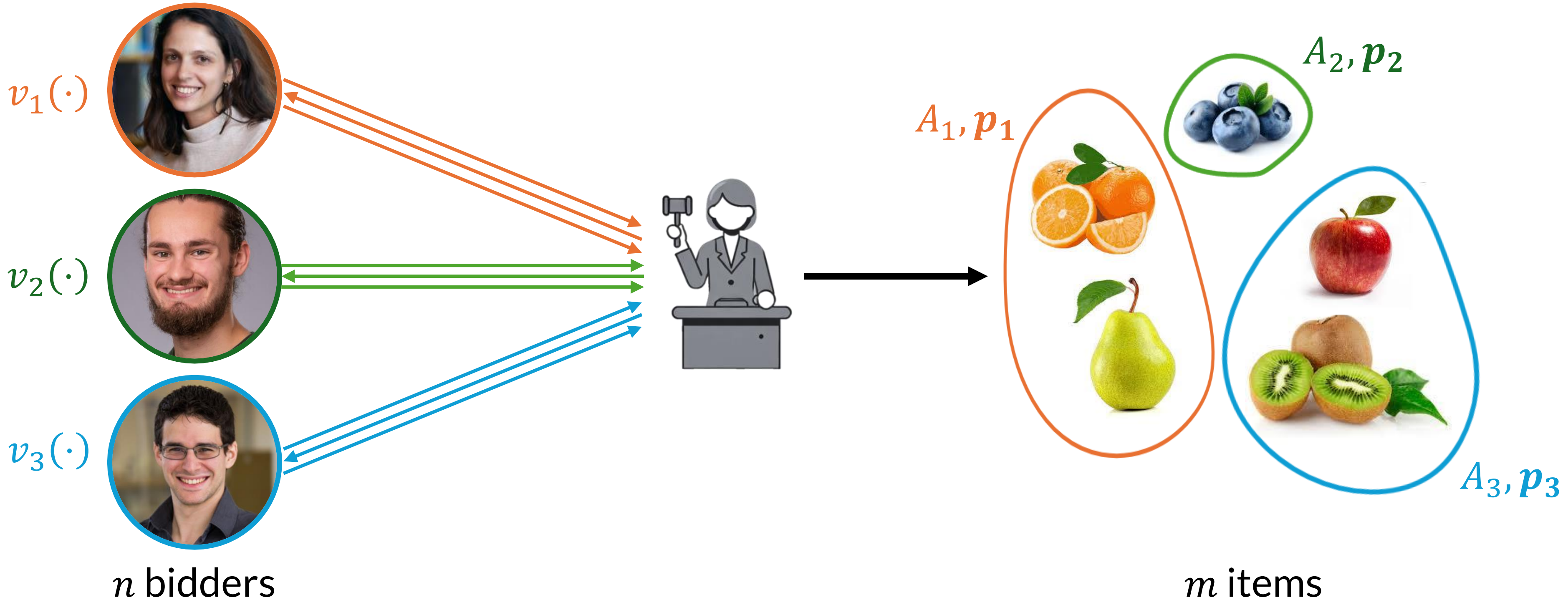


- n-player deterministic blackboard communication
- **Communication cost**: maximum number of bits sent in the worst case

Problem setup: truthfulness



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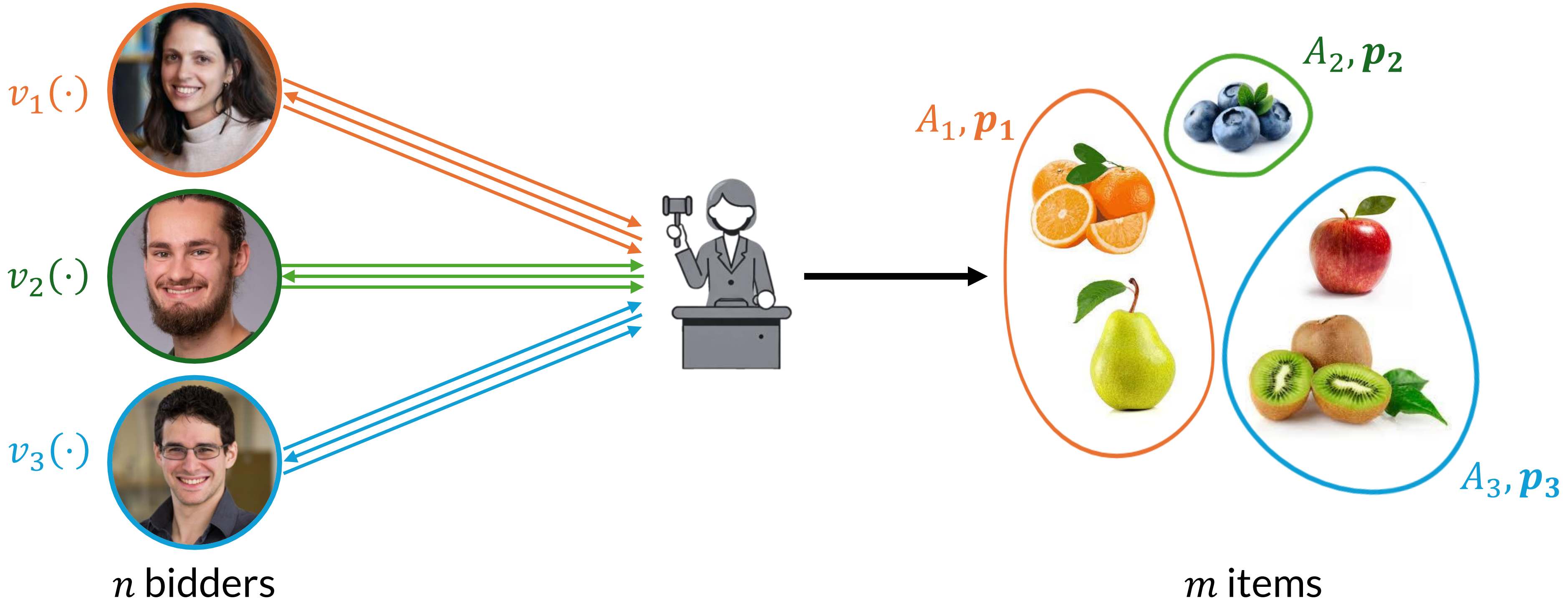

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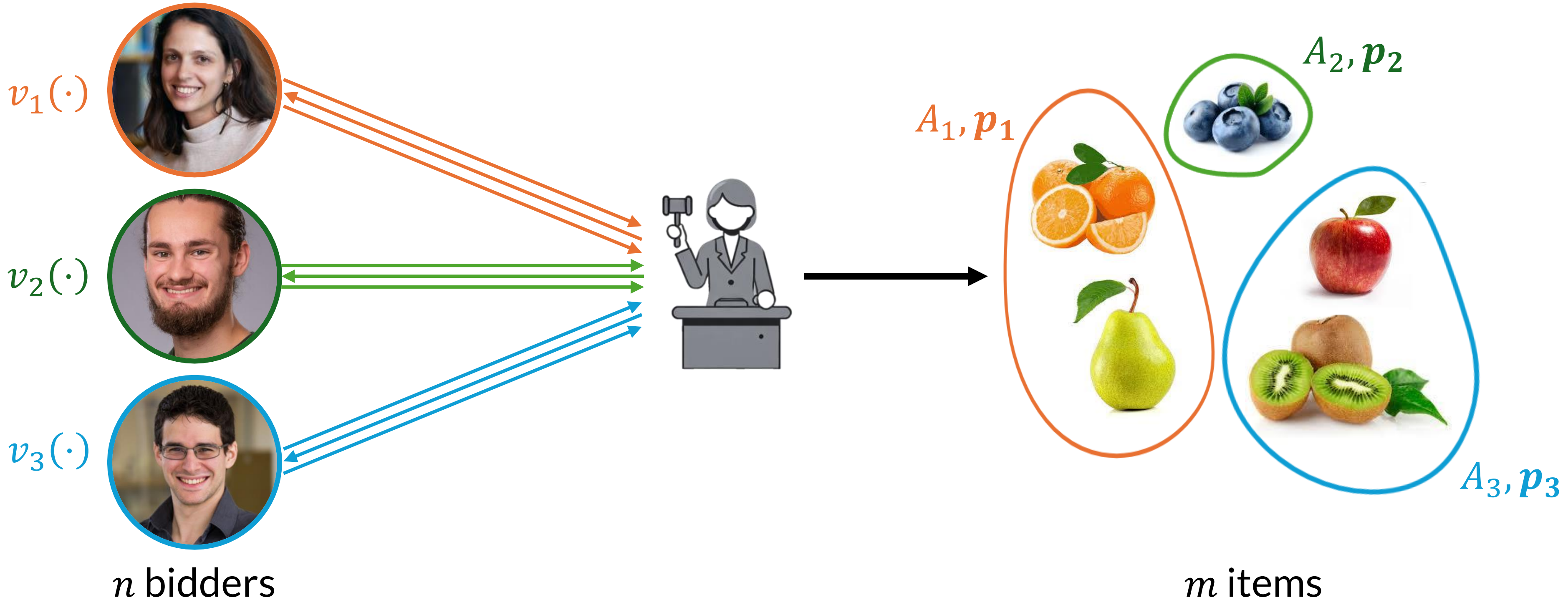

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  (Truth-telling is *ex-post Nash equilibrium*)

Existing results

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- Everyone sends v_i, then run VCG mechanism [Vickrey 61, Clarke 71, Groves 73]

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(a) & **(b)** still poorly understood

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- Huge gap for truthful vs. non-truthful protocols

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- Huge gap for truthful vs. non-truthful protocols
- It remains unknown **whether there should be any gap at all**!

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       - Heavily relies on the two-bidder corollary

Question: is there any gap *when* $n \geq 3$?

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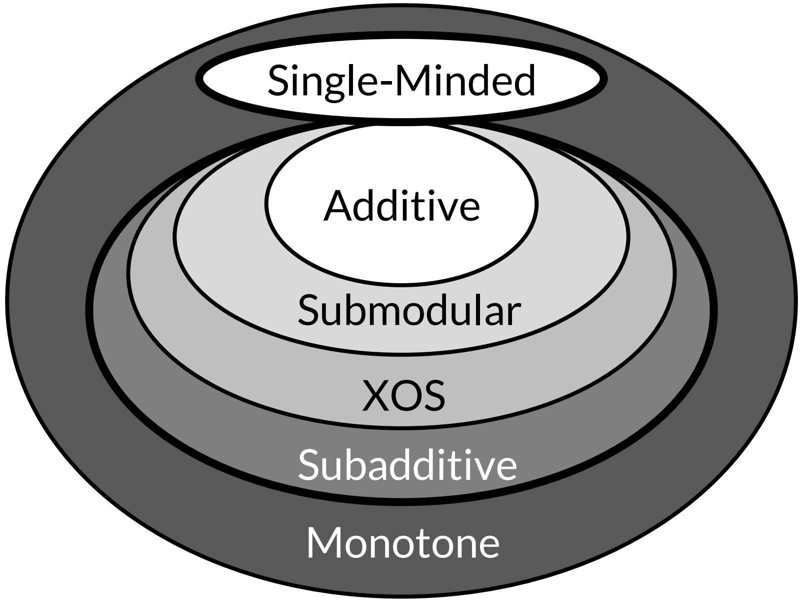
- **Main result**: Separation for 3 bidders that are either **Subadditive** or **Single-Minded**



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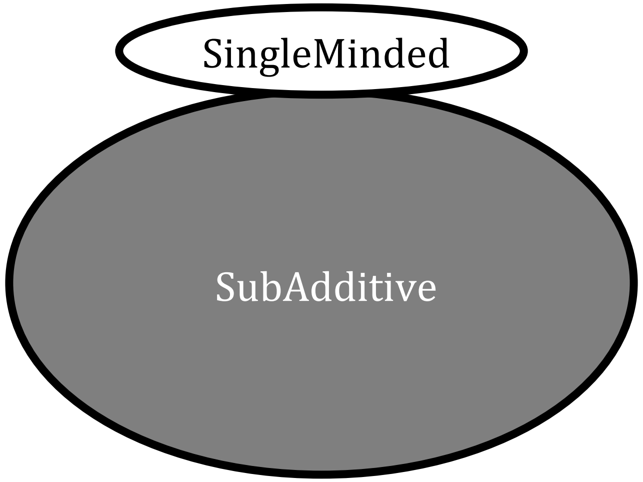

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  - **Impossibility of $\mathbf{0.366}$-approximation efficient & truthful mechanisms**

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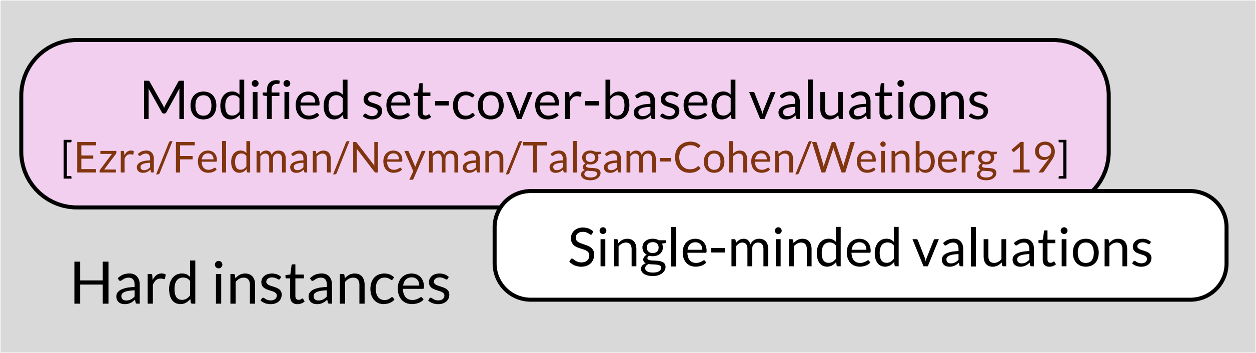
  - **Impossibility of $\mathbf{0.366}$-approximation efficient & truthful mechanisms**
  - Existence of 0.5-approximation efficient non-truthful protocols

Overview of approach



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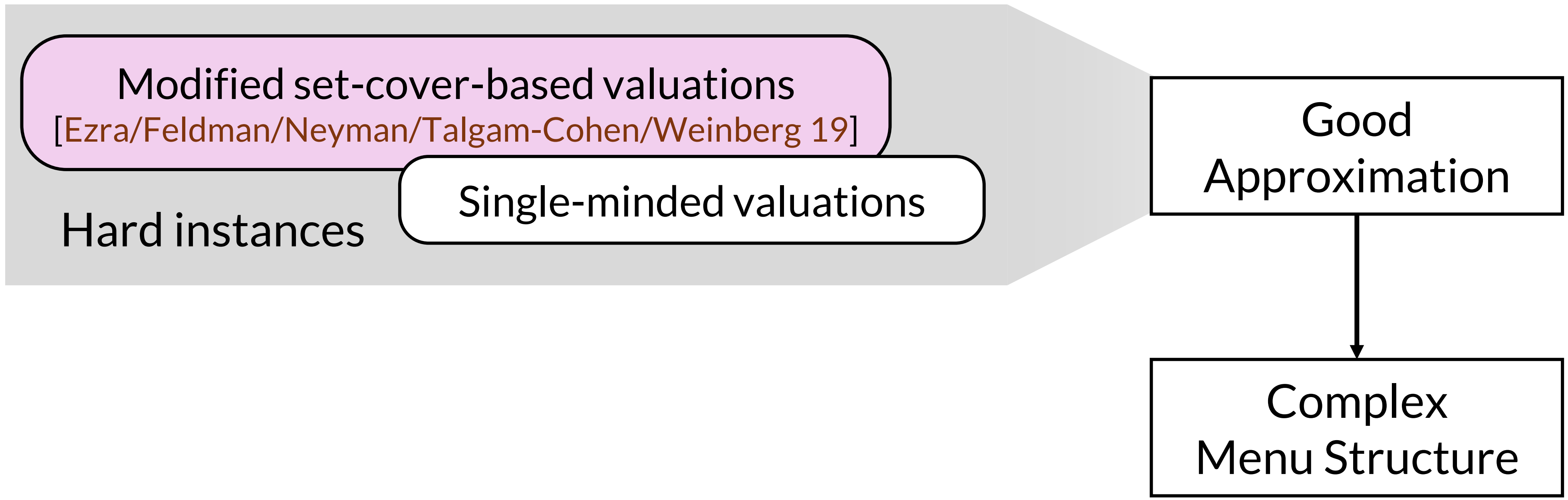

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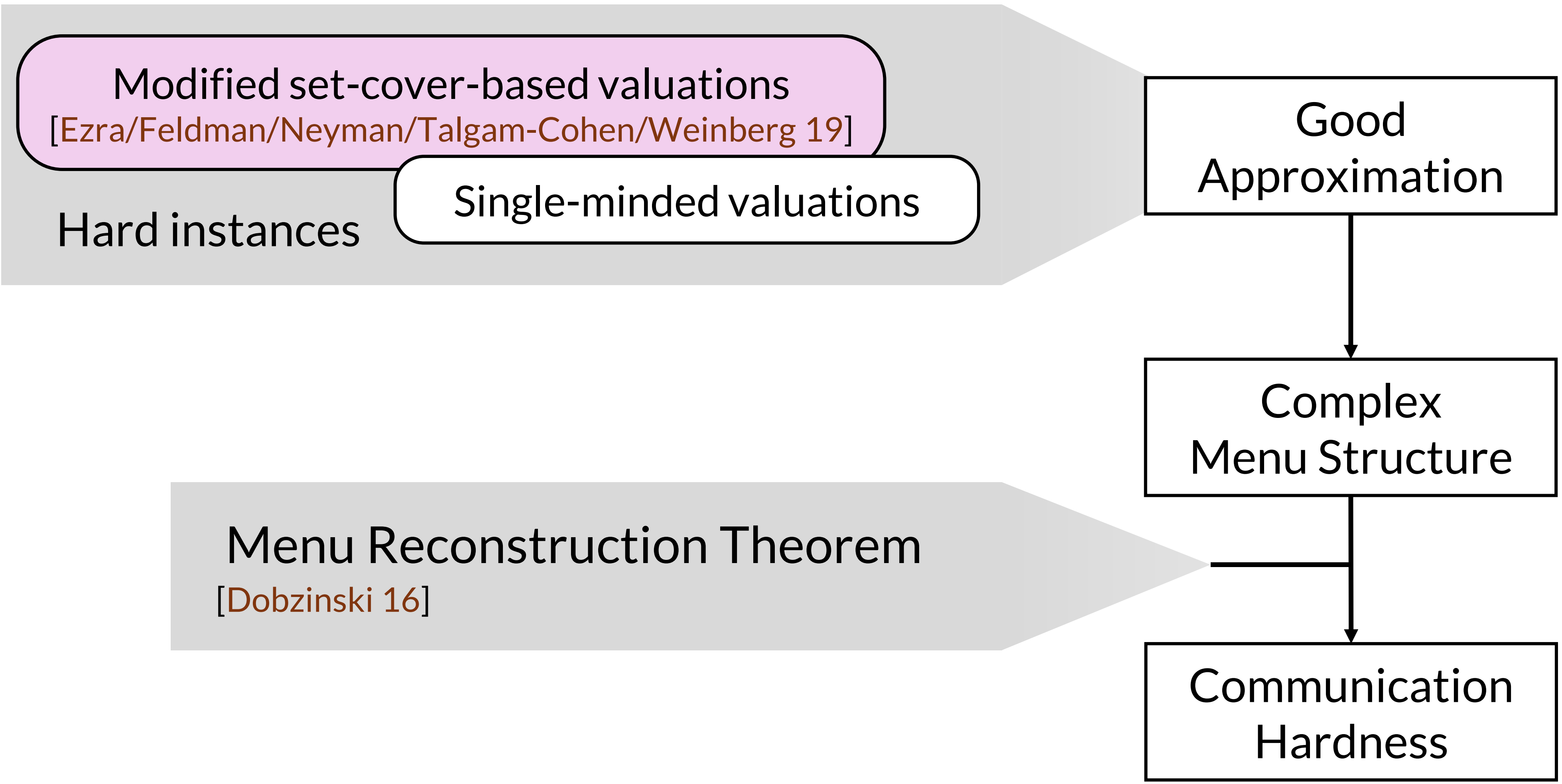
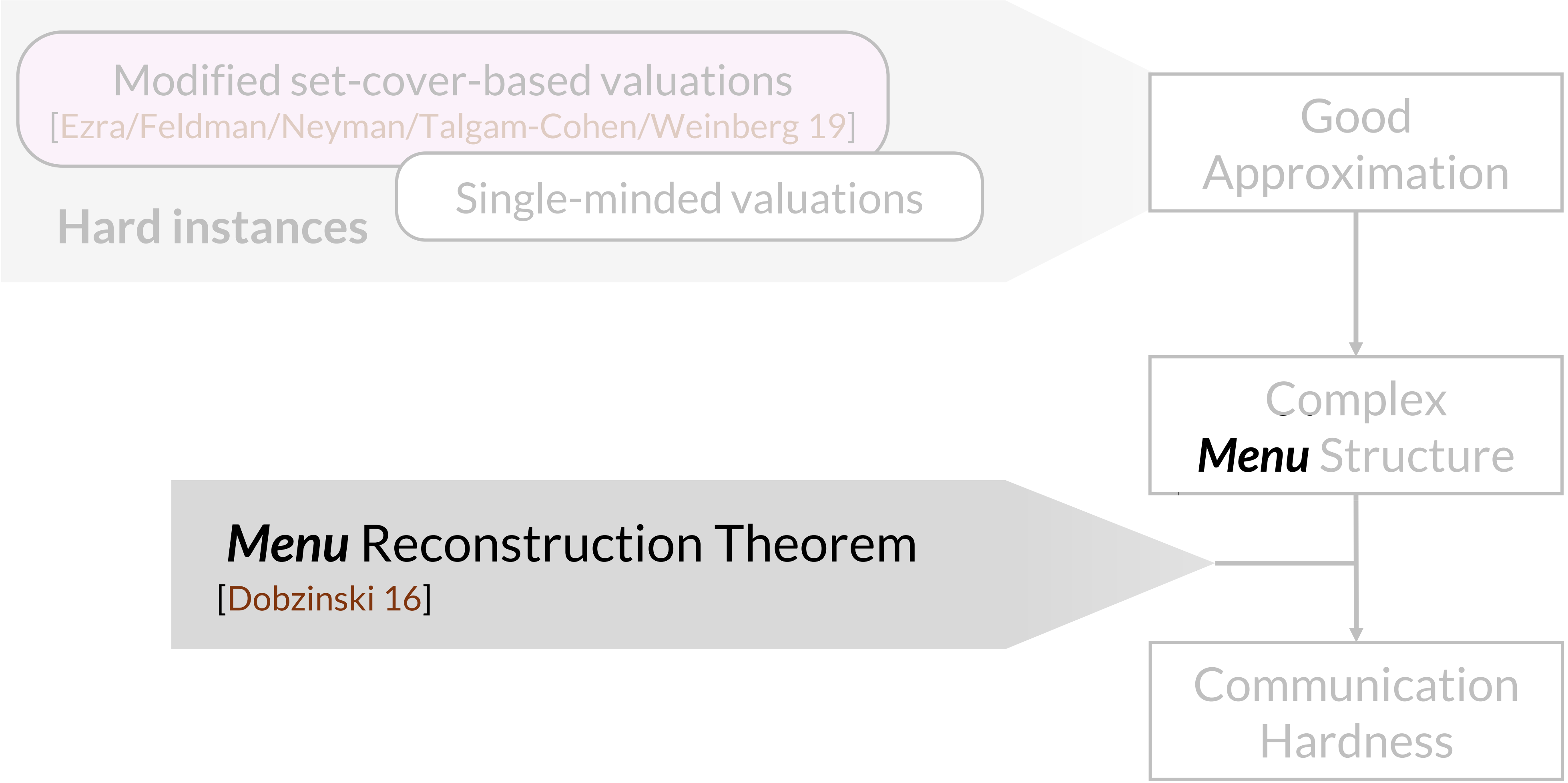
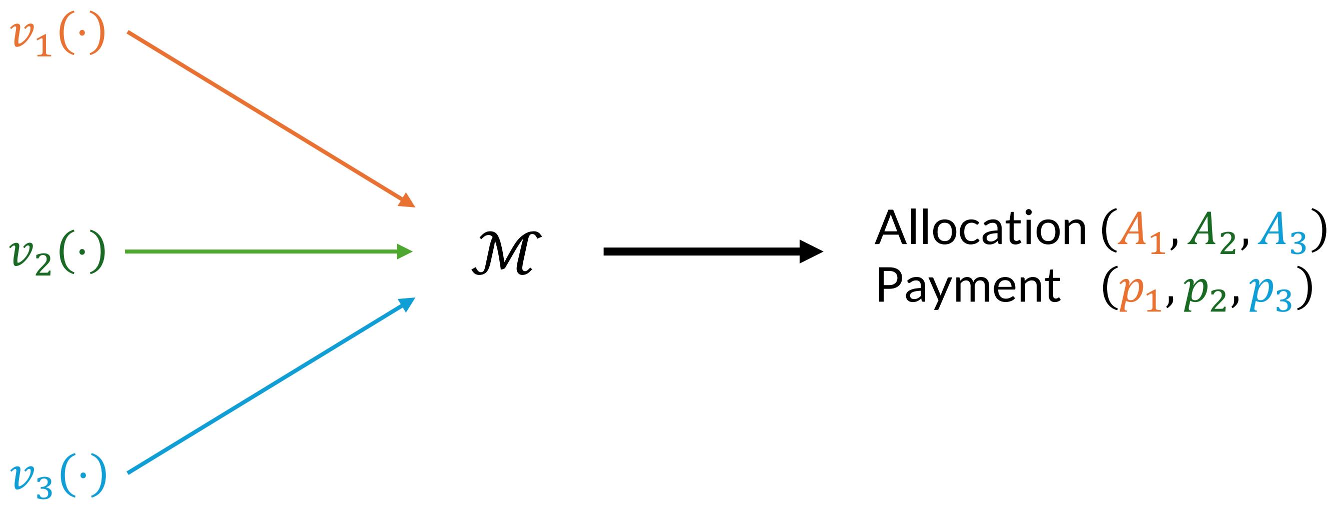


Key concept: menus

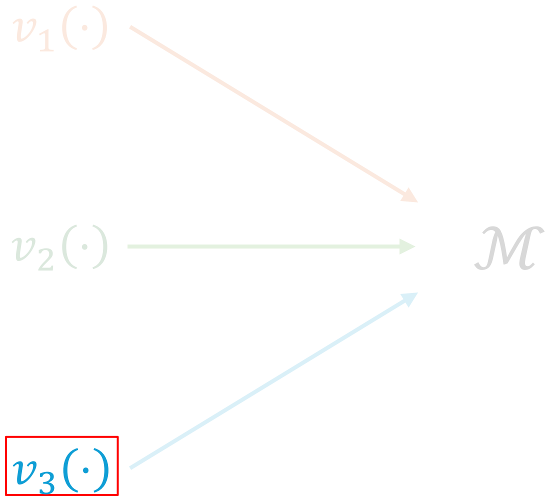


Key concept: menu for $\mathcal{M}$

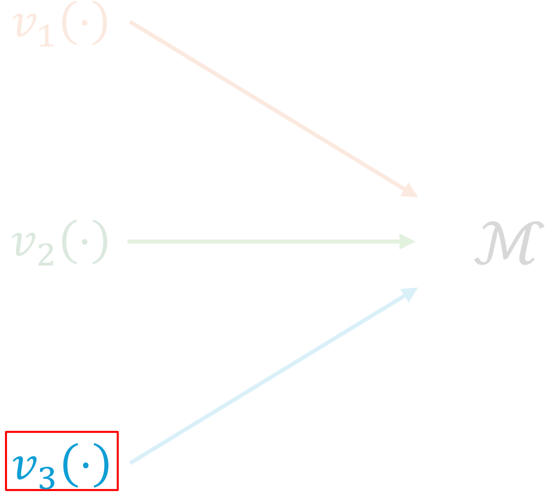


Truthful mechanism $\mathcal{M}$

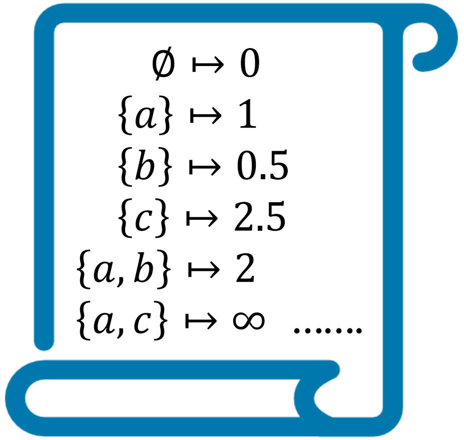
Key concept: menu for $\mathcal{M}$ and bidder i



Key concept: menu for $\mathcal{M}$ and bidder i

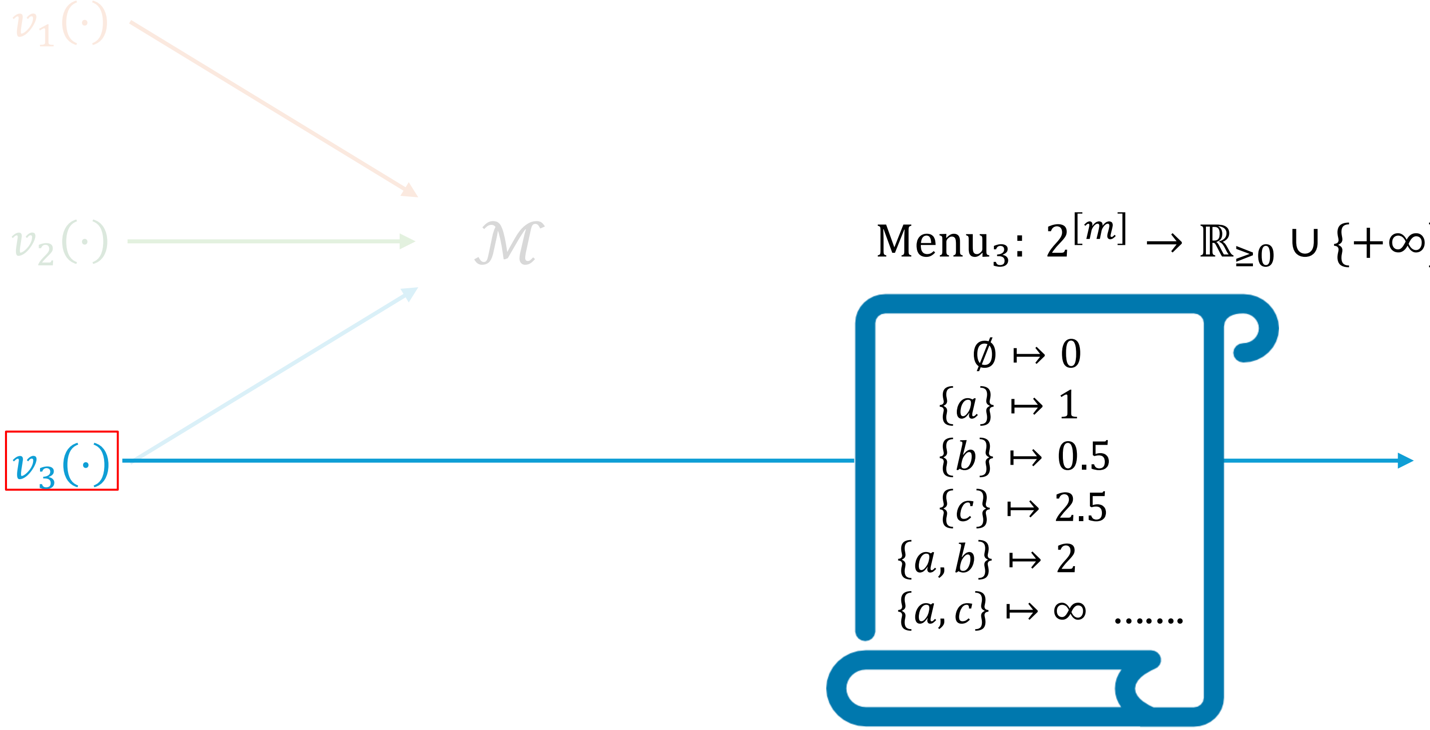


$\text{Menu}_3\colon\ 2^{[m]} \to \mathbb{R}_{\geq 0} \cup \{+\infty\}$



Key concept: menu for $\mathcal{M}$ and bidder i

$\text{Menu}_3 : 2^{[m]} \to \mathbb{R}_{\geq 0} \cup \{+\infty\}$



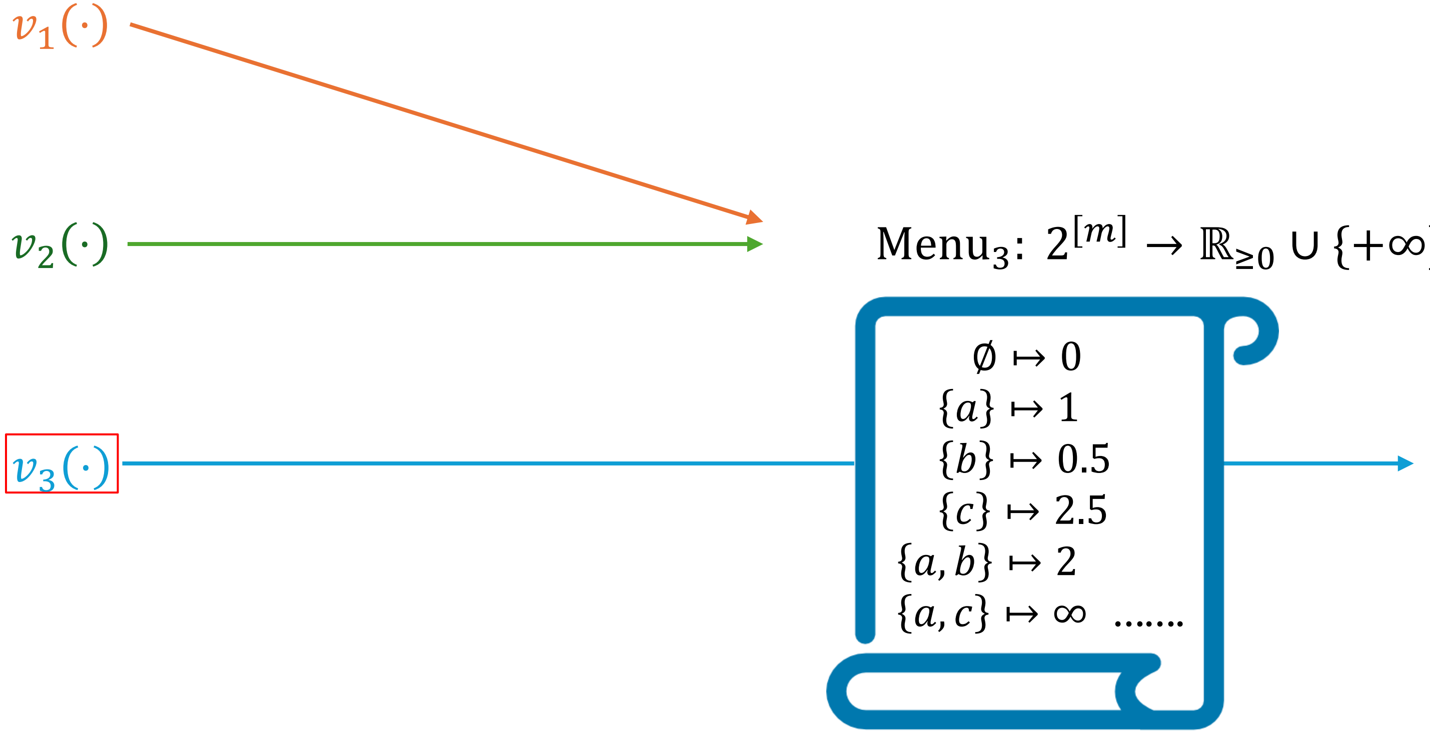
Pick *utility-maximizing* bundle

$A^* = \arg\max_S v_3(S) - \text{Menu}_3(S)$

with payment

$p^* = \text{Menu}_3(A^*)$

Key concept: menu for $\mathcal{M}$ and bidder i



Pick *utility-maximizing* bundle

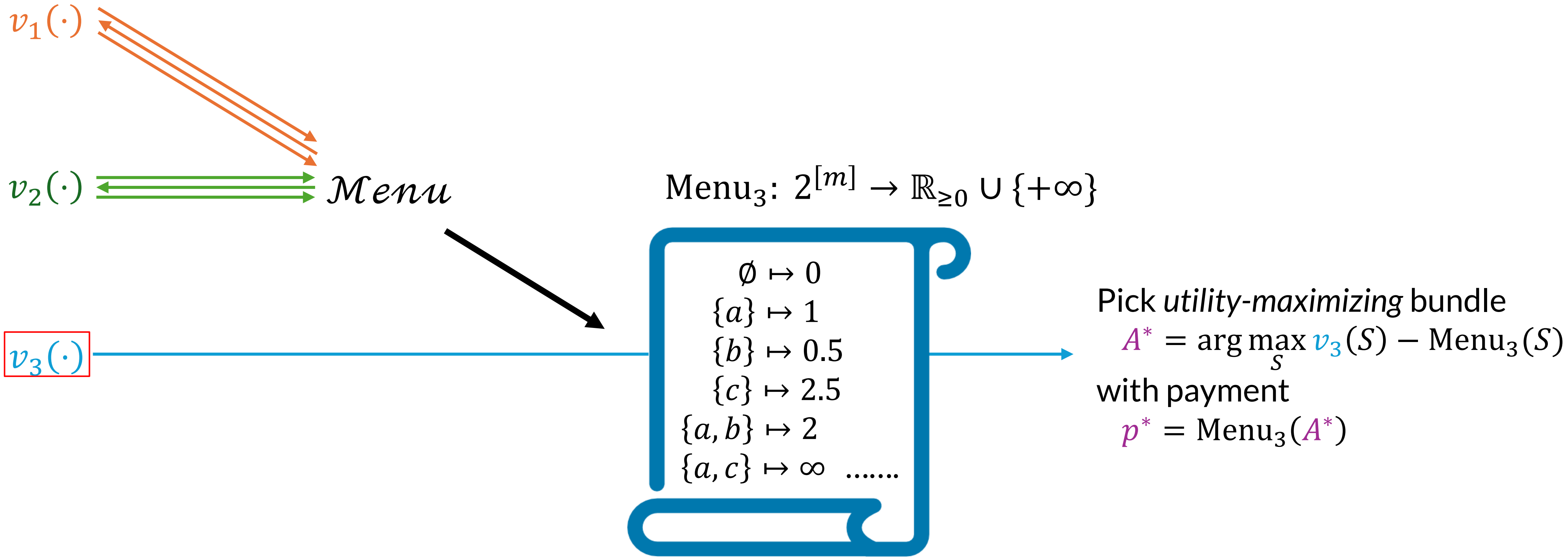
$A^* = \arg\max_S v_3(S) - \text{Menu}_3(S)$

with payment

$p^* = \text{Menu}_3(A^*)$

Taxation Principle [Hammond 79, Guesnerie 81]: $\forall\, v_{-i}, \exists\, \text{Menu}_i$ with identical results as $\mathcal{M}$

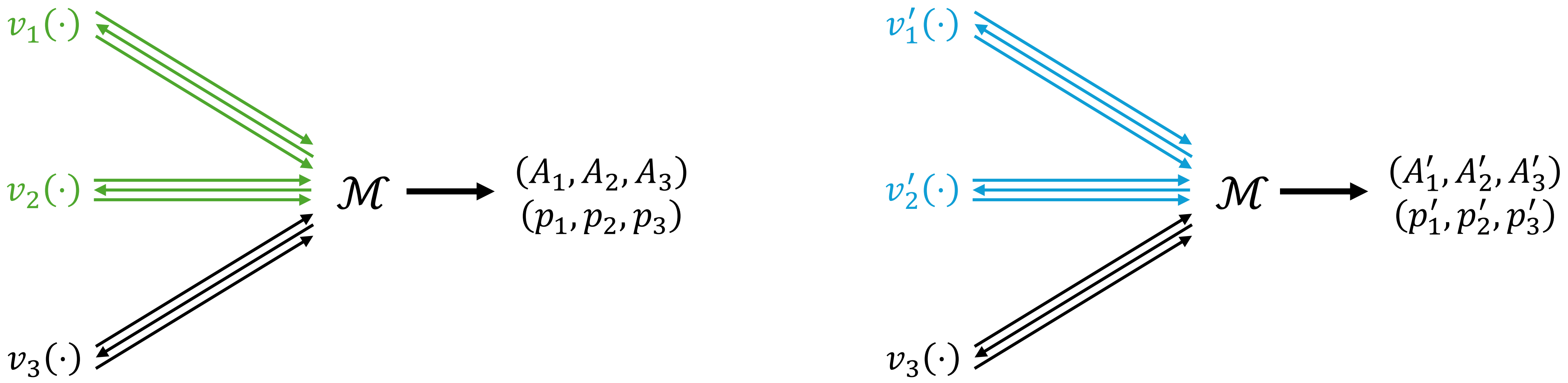
Key concept: menu for $\mathcal{M}$ and bidder i



Taxation Principle [Hammond 79, Guesnerie 81]: $\forall\, v_{-i}, \exists\, \text{Menu}_i$ with identical results as $\mathcal{M}$

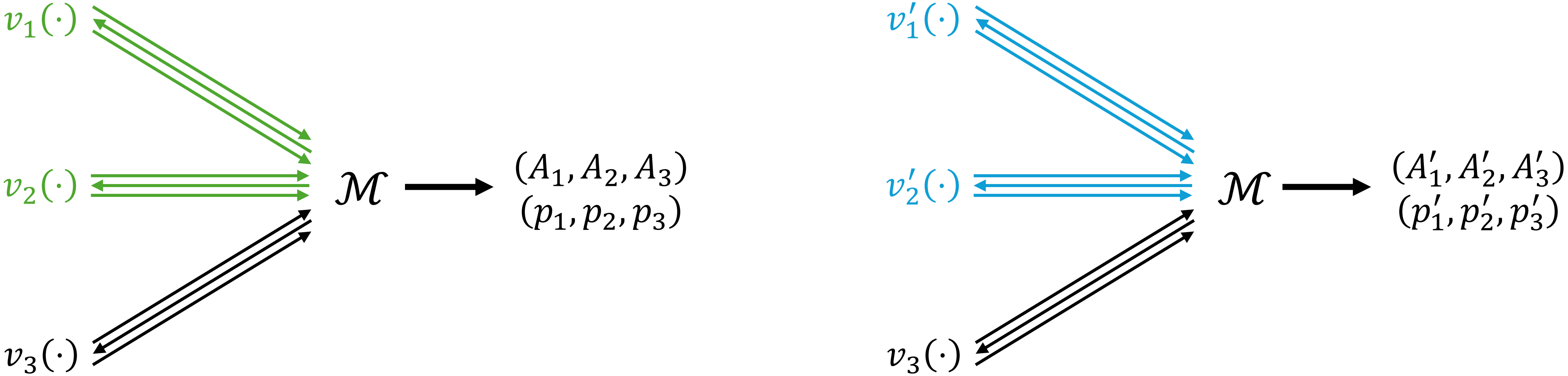
Menu Reconstruction Theorem [Dobzinski 16]: If $\mathcal{M}$ can be implemented efficiently, Menu_i can be reconstructed efficiently from v_{-i} (with some caveats)

Example

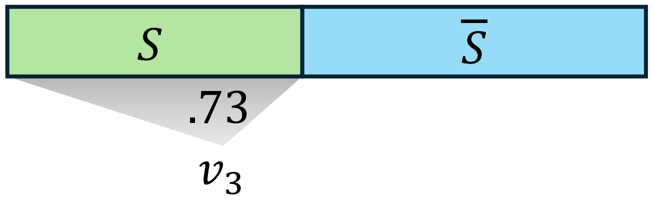


- Two instances (v_1, v_2, v_3), (v'_1, v'_2, v_3)

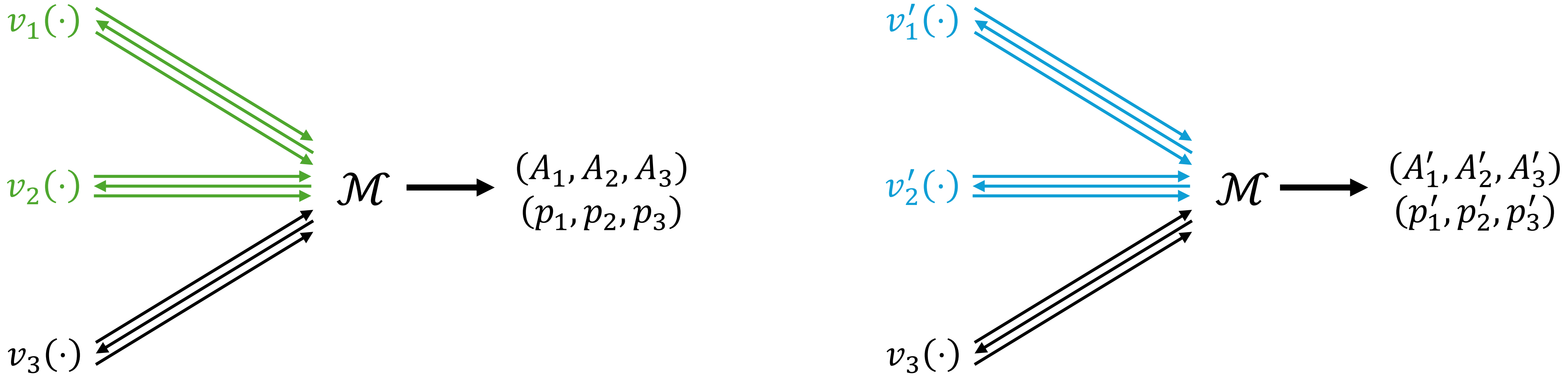
Example



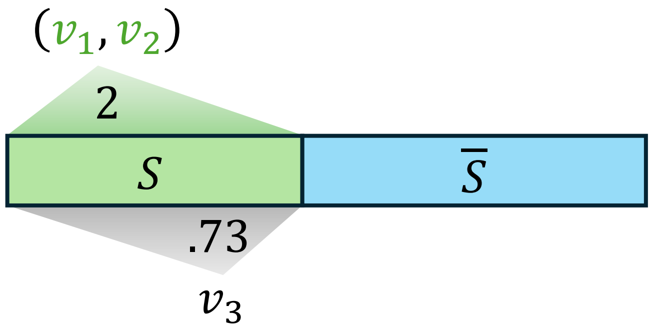
- Two instances (v_1, v_2, v_3), (v_1', v_2', v_3)
  - v_3 only interested in S with value ≈ 0.73



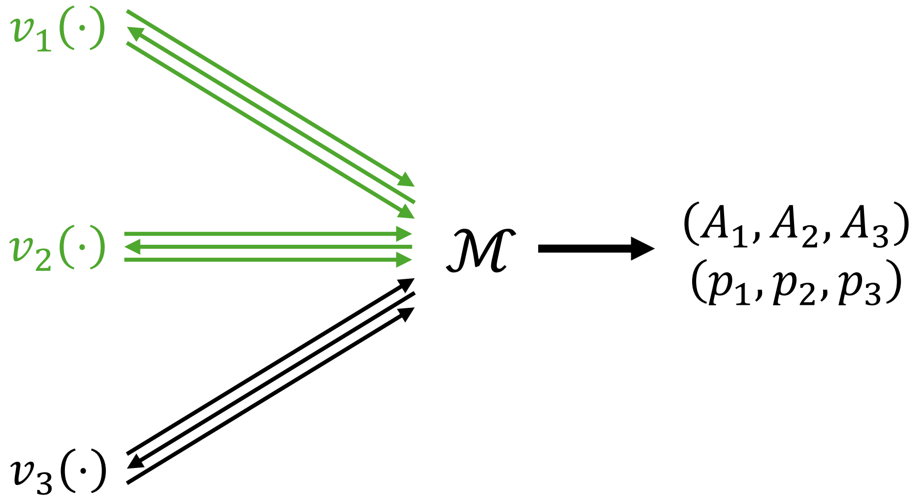
Example

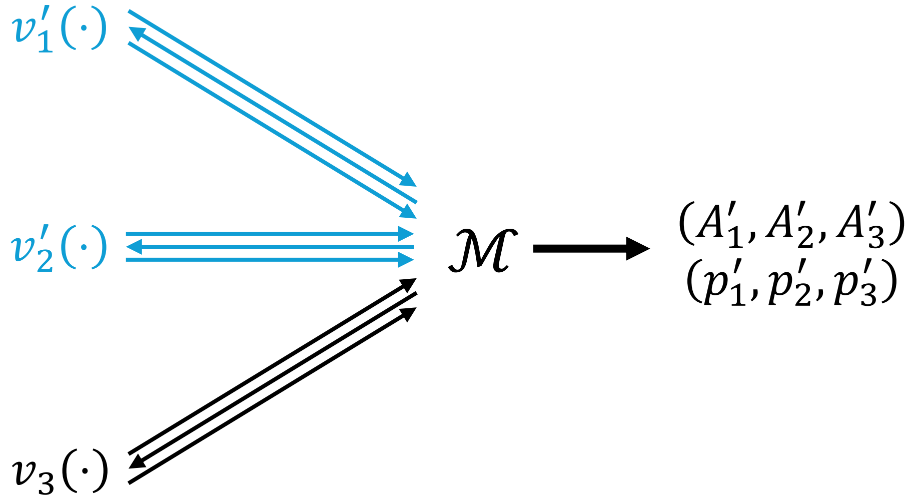


- Two instances (v_1, v_2, v_3), (v_1', v_2', v_3)
  - (v_1, v_2) gets total welfare 2 from S only
  - v_3 only interested in S with value ≈ 0.73

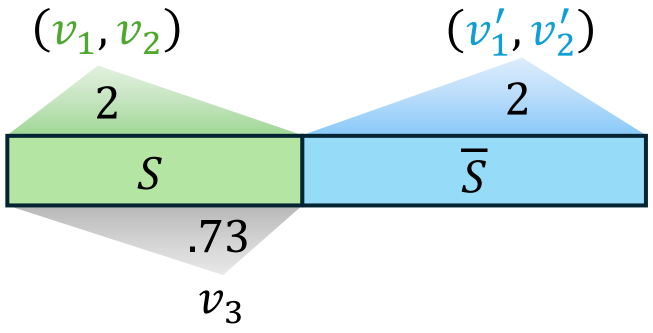


Example

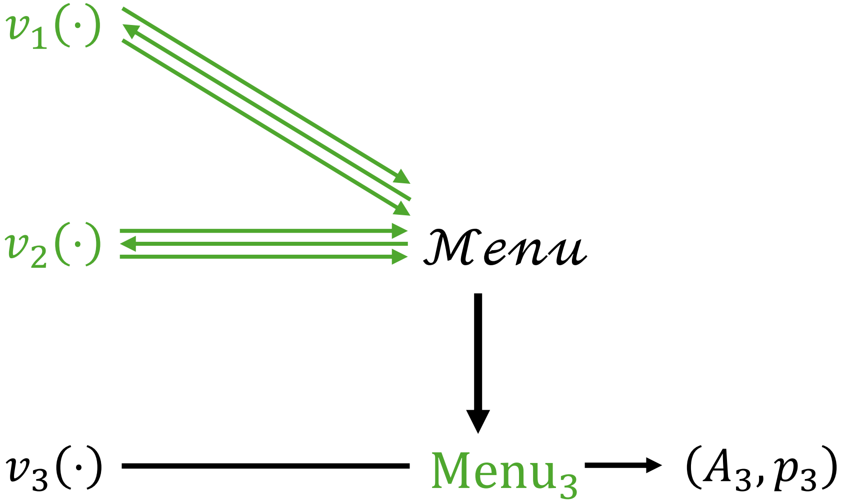


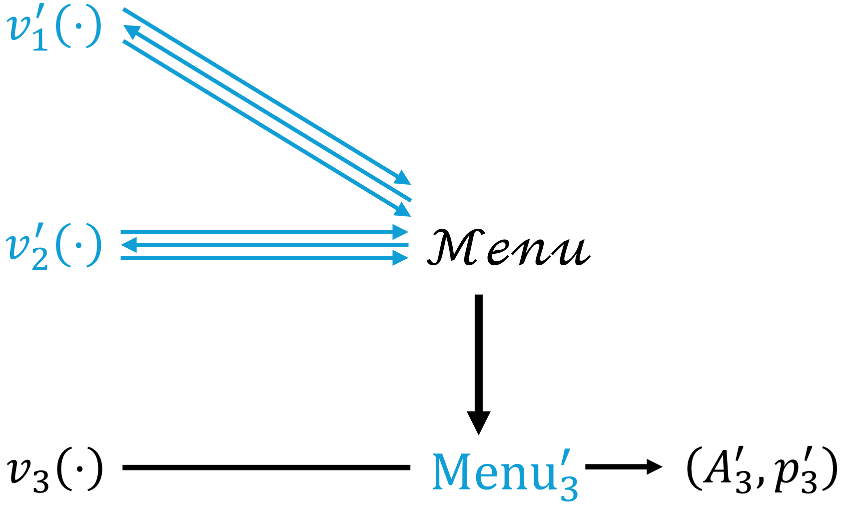


- Two instances (v_1, v_2, v_3), (v'_1, v'_2, v_3)
  - (v_1, v_2) gets total welfare 2 from S only
  - (v'_1, v'_2) gets total welfare 2 from $\overline{S}$ only
  - v_3 only interested in S with value ≈ 0.73

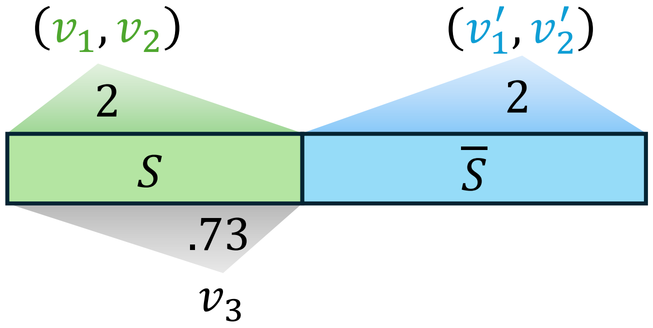


Example

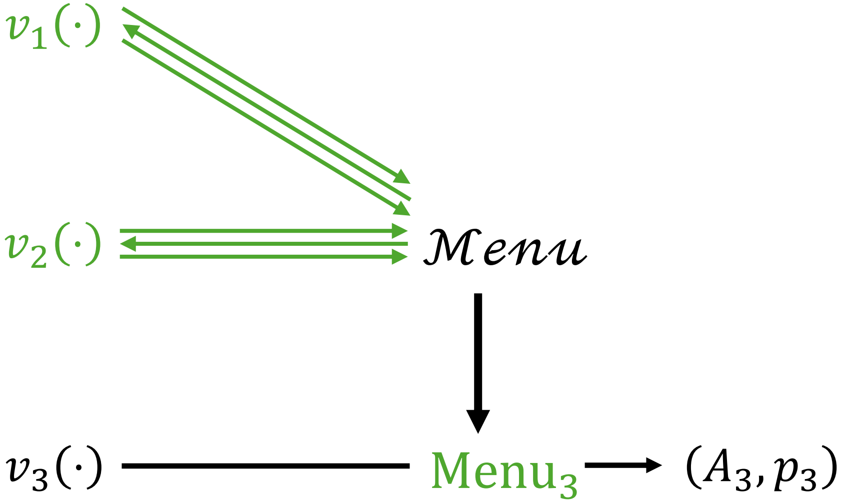


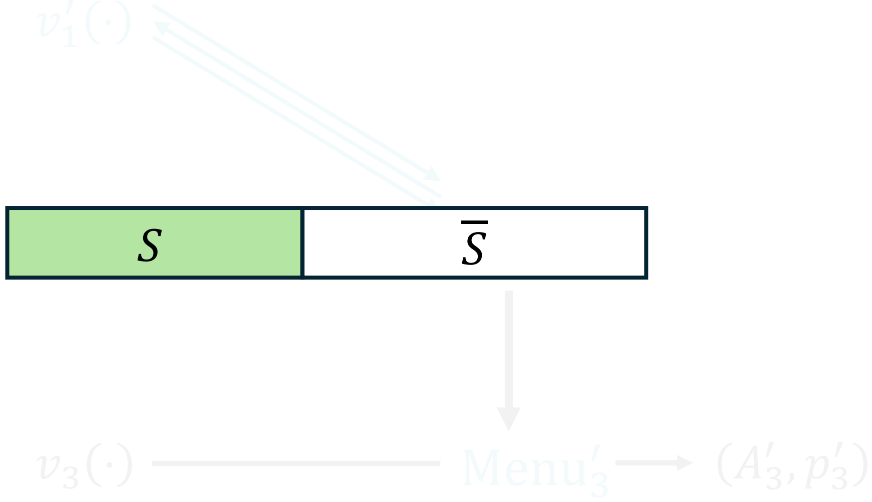


- Two instances (v_1, v_2, v_3), (v_1', v_2', v_3)
  - (v_1, v_2) gets total welfare 2 from S only
  - (v_1', v_2') gets total welfare 2 from $\overline{S}$ only
  - v_3 only interested in S with value ≈ 0.73

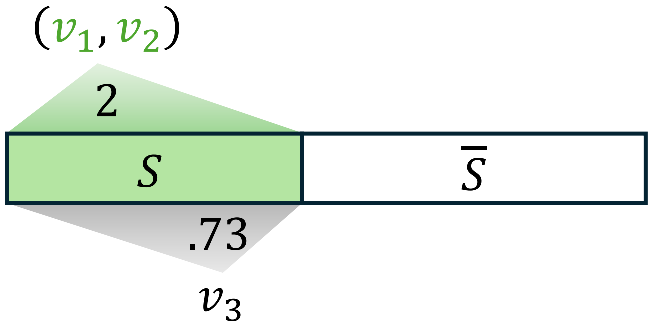


Example: when S is cheap

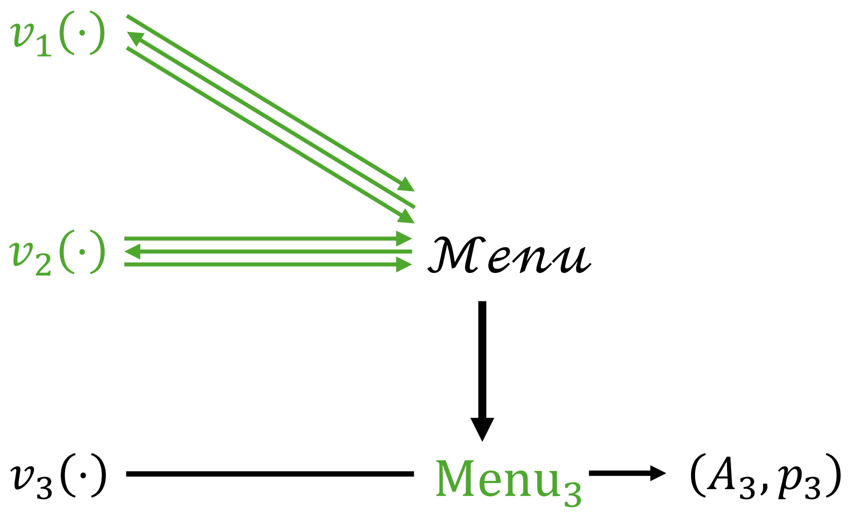


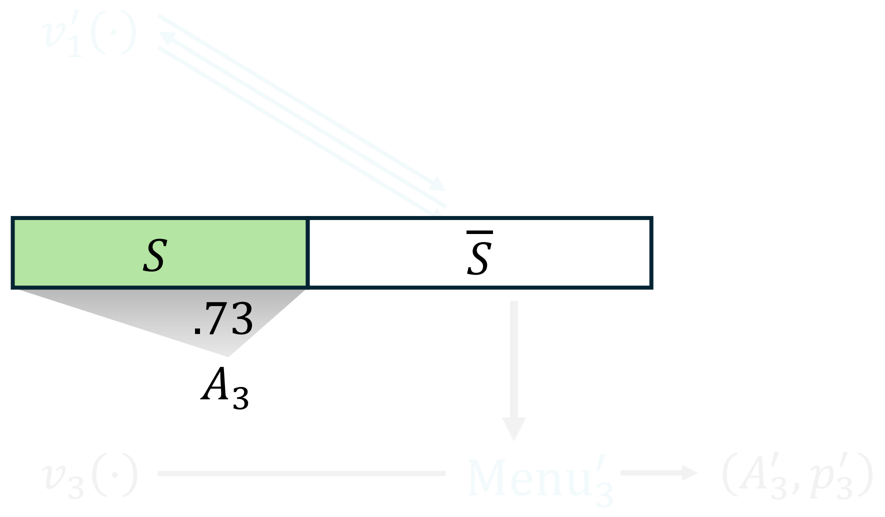


- When $\text{Menu}_3(S) \leq 0.73$

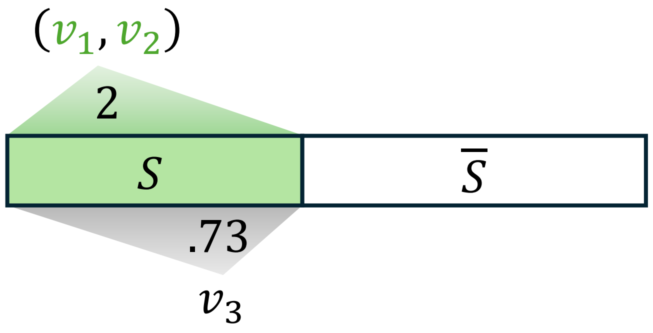


Example: when S is cheap

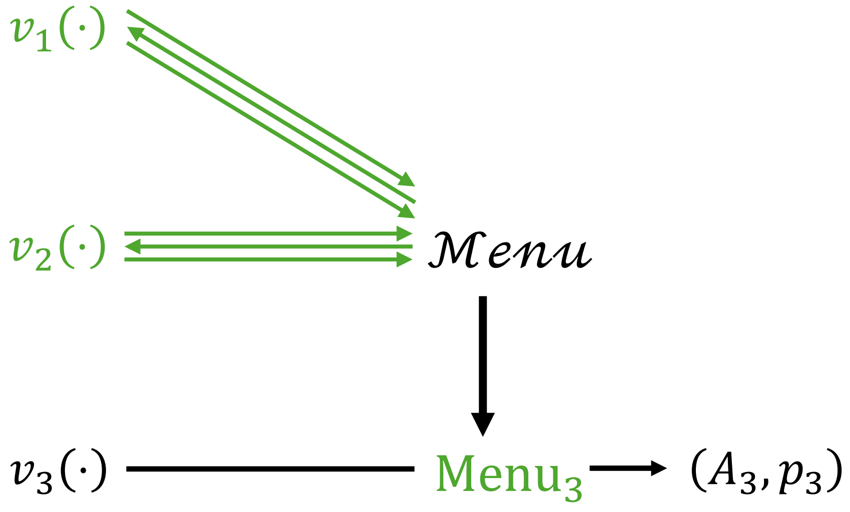


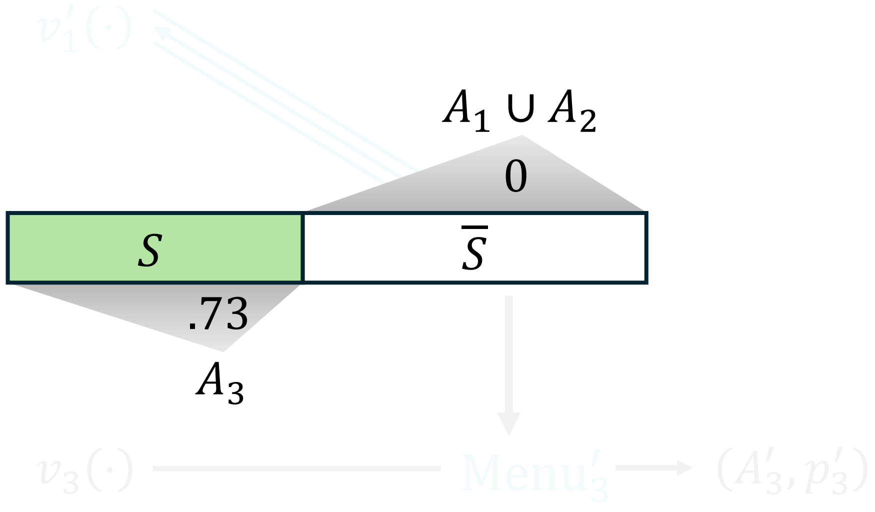


- When $\text{Menu}_3(S) \leq 0.73$
  - $A_3 = S$

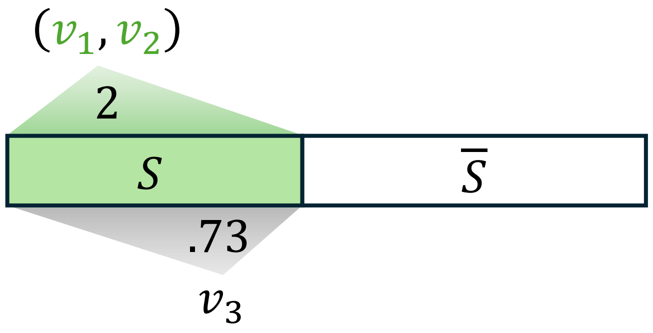


Example: when S is cheap

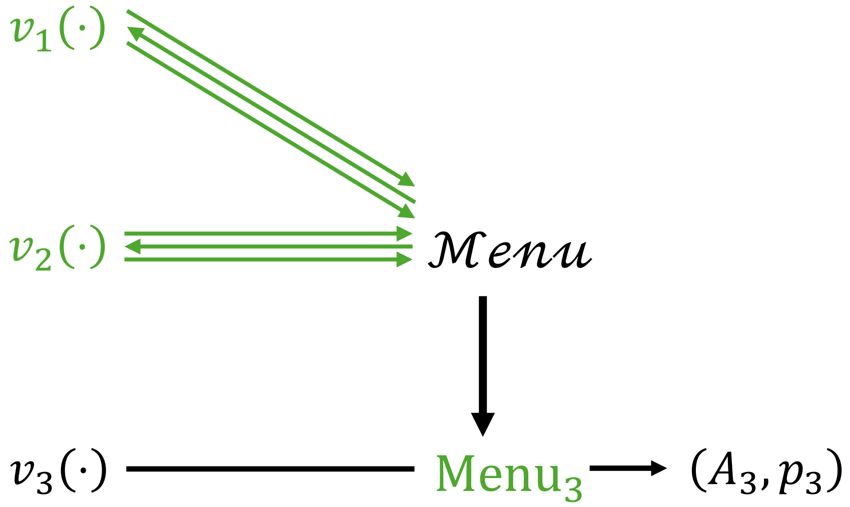


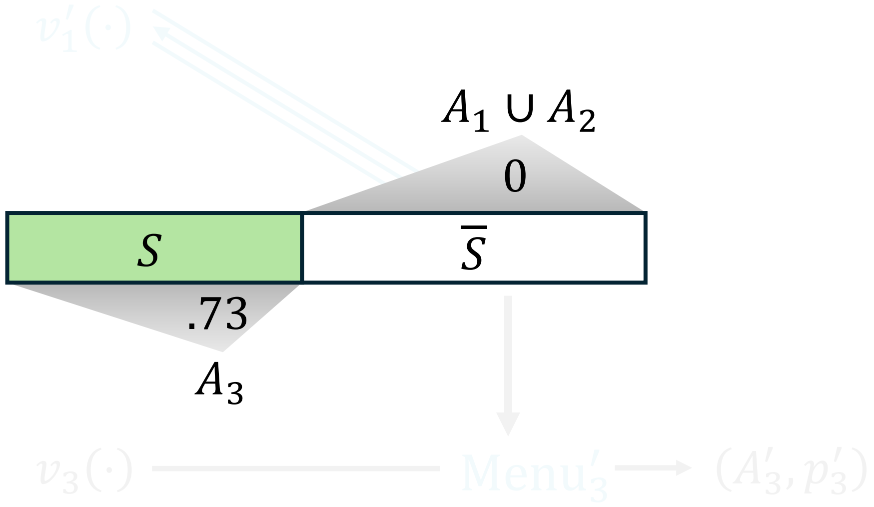


- When $\text{Menu}_3(S) \leq 0.73$
  - $A_3 = S$
  - $A_1 \cup A_2 \subseteq \bar{S}$



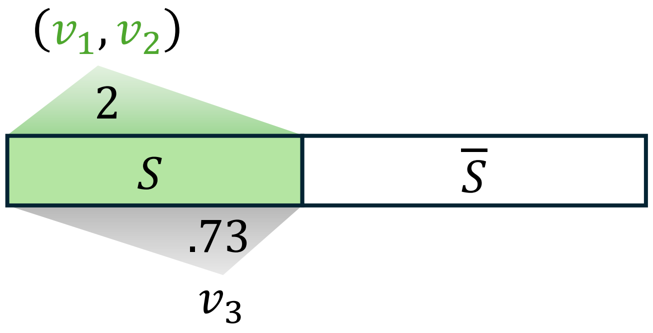
Example: when S is cheap



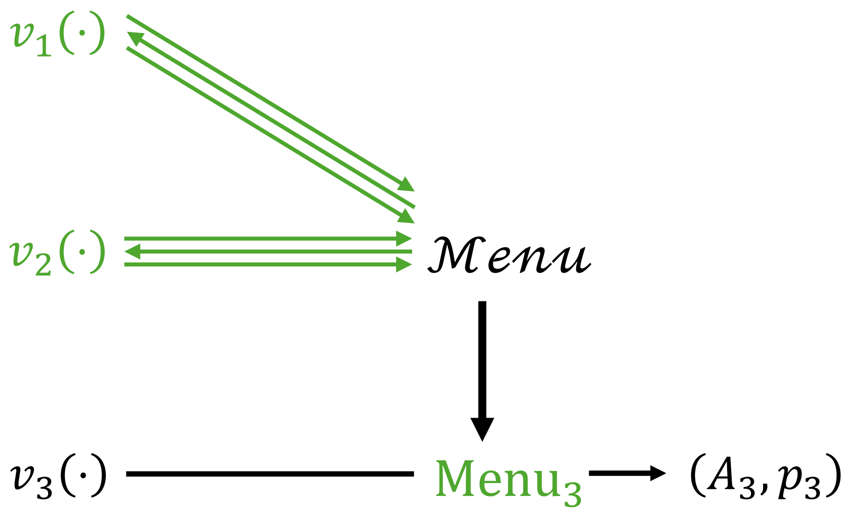


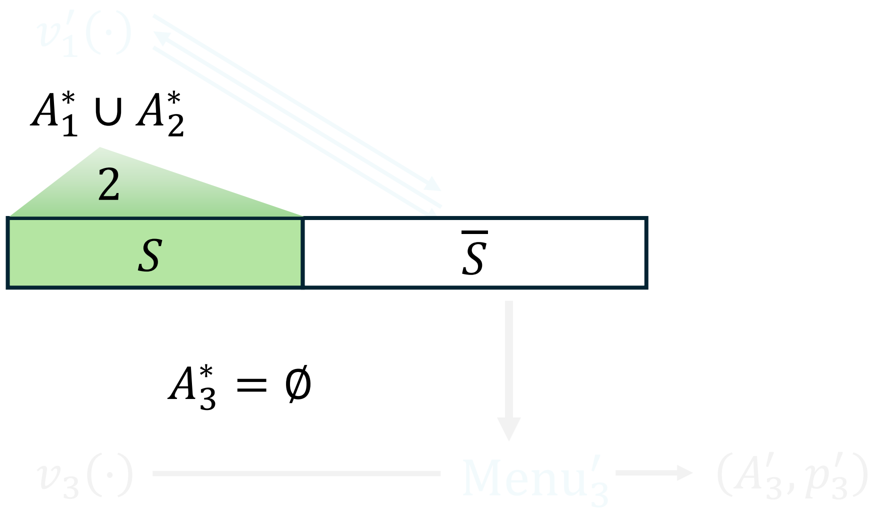
- When $\text{Menu}_3(S) \leq 0.73$
  - $A_3 = S$
  - $A_1 \cup A_2 \subseteq \overline{S}$

  } welfare = 0.73



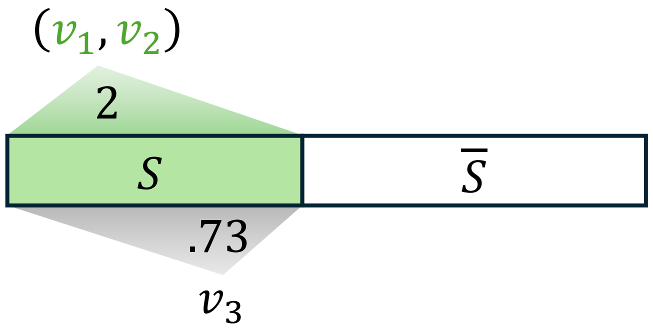
Example: when S is cheap





- When $\text{Menu}_3(S) \leq 0.73$
  - $A_3 = S$
  - $A_1 \cup A_2 \subseteq \overline{S}$

  } welfare = 0.73
  - However, optimal A_1^*, A_2^*, A_3^* give welfare 2



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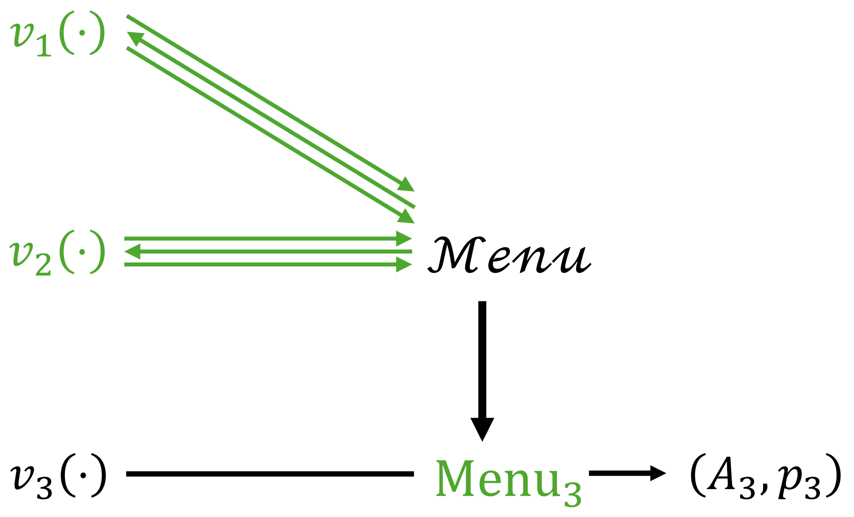

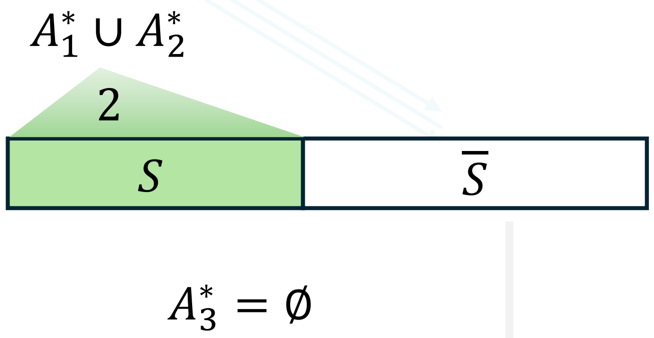

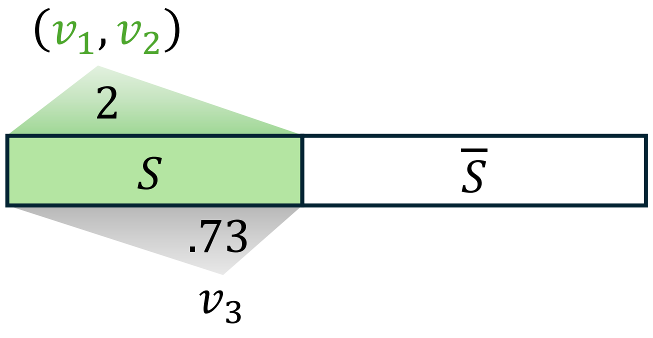

- When $\text{Menu}_3(S) \leq 0.73$
  - $A_3 = S$
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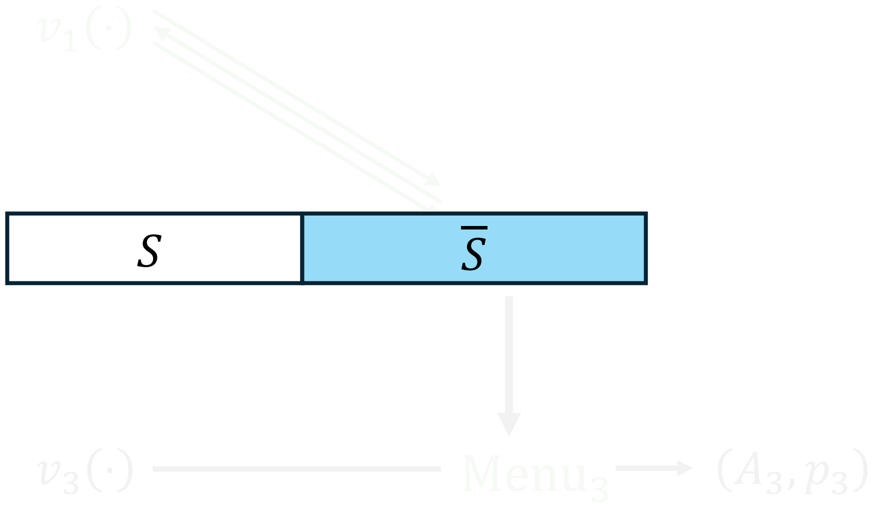
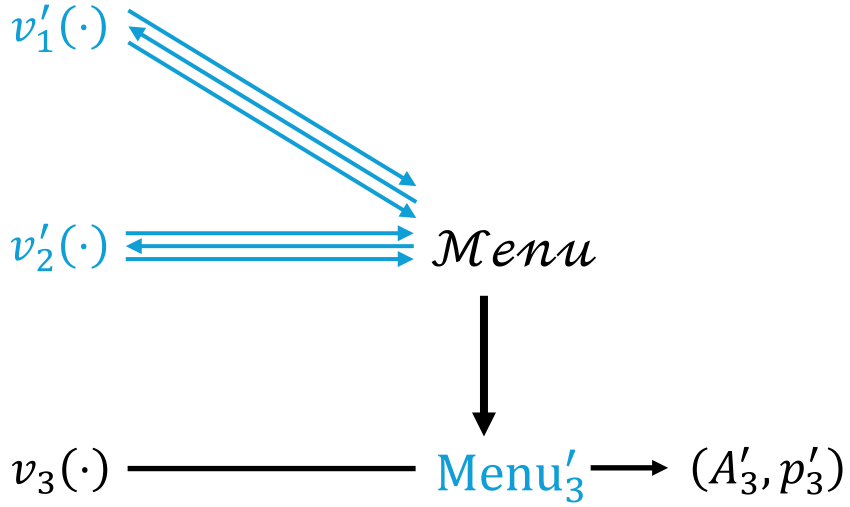
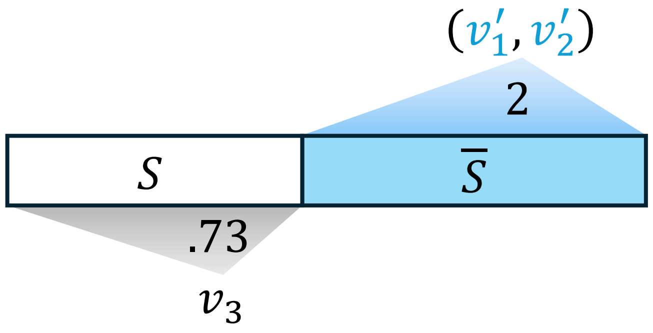
  } welfare = 0.73
  - However, optimal A_1^*, A_2^*, A_3^* give welfare 2
  - 0.366−approximation!

Example: when S is expensive

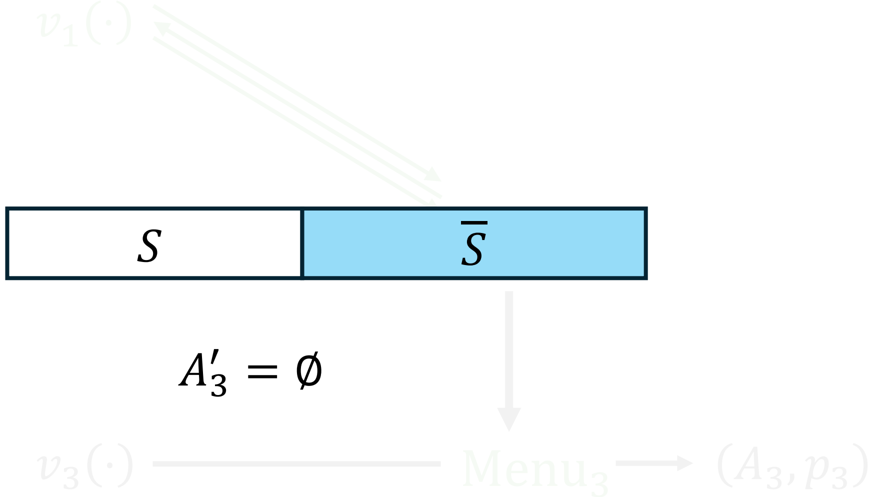


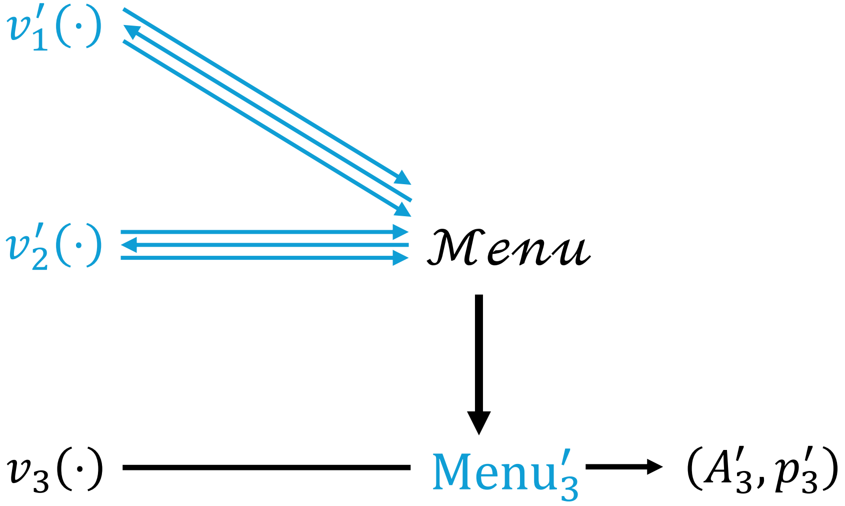


- When $\mathrm{Menu}_3'(S) > 0.73$

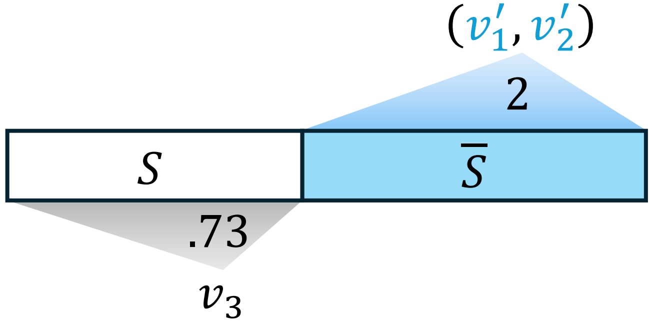


Example: when S is expensive

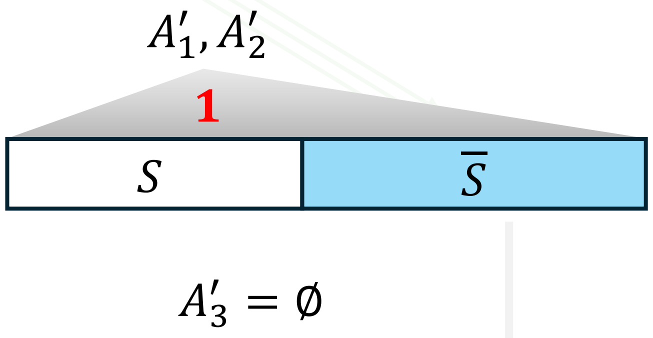


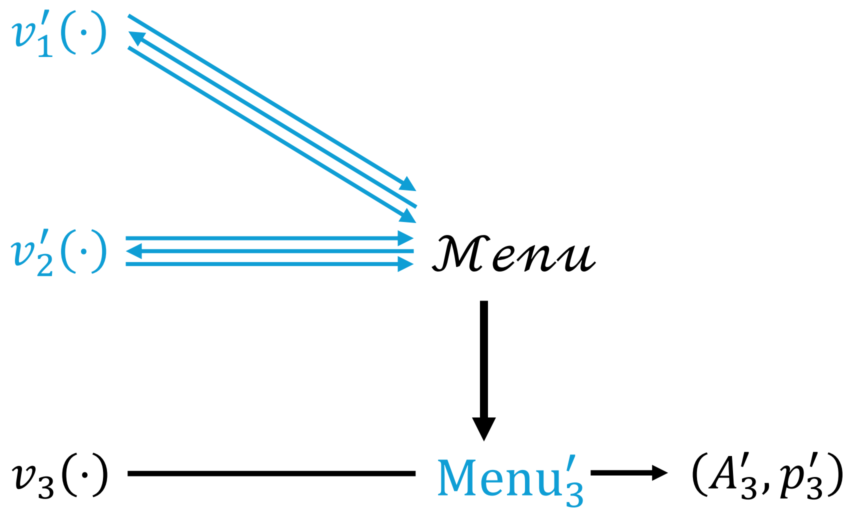


- When $\text{Menu}'_3(S) > 0.73$
  - $A'_3 = \emptyset$



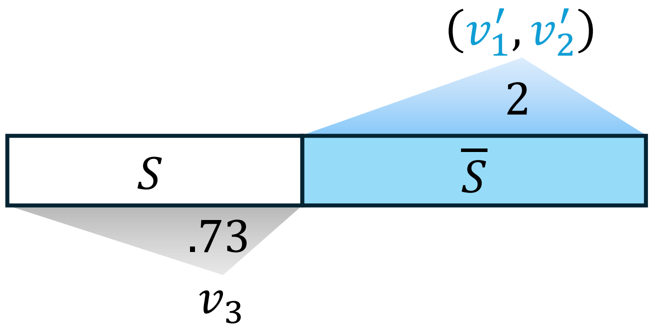
Example: when S is expensive





- When $\text{Menu}'_3(S) > 0.73$
  - $A'_3 = \emptyset$
  - $A'_1 \cup A'_2 \subseteq [m]$

  } welfare = **1**



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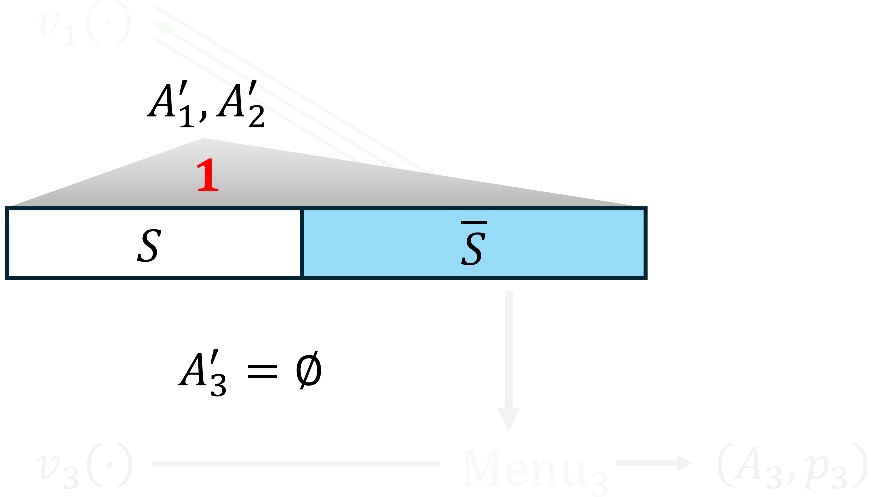

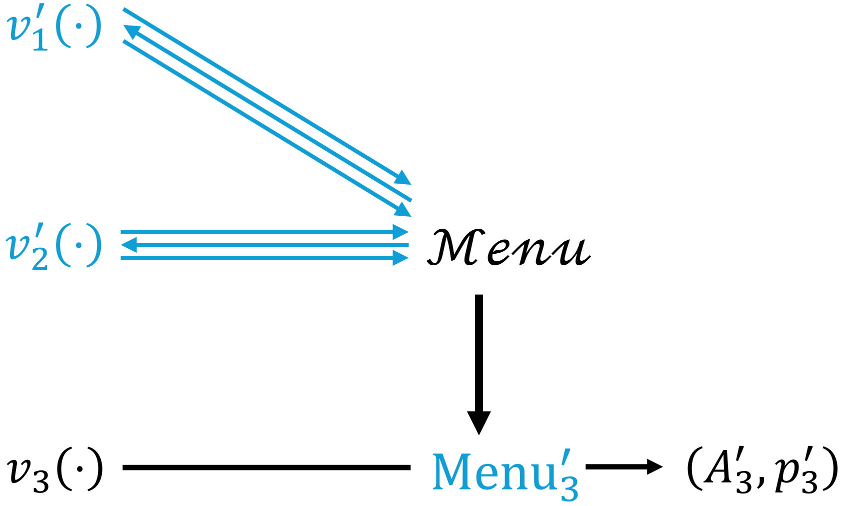

- When $\text{Menu}'_3(S) > 0.73$
  - $A'_3 = \emptyset$
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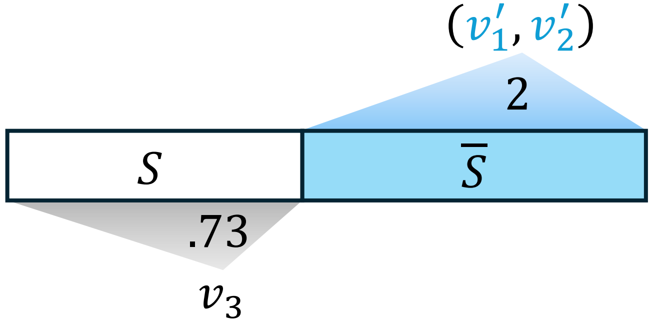
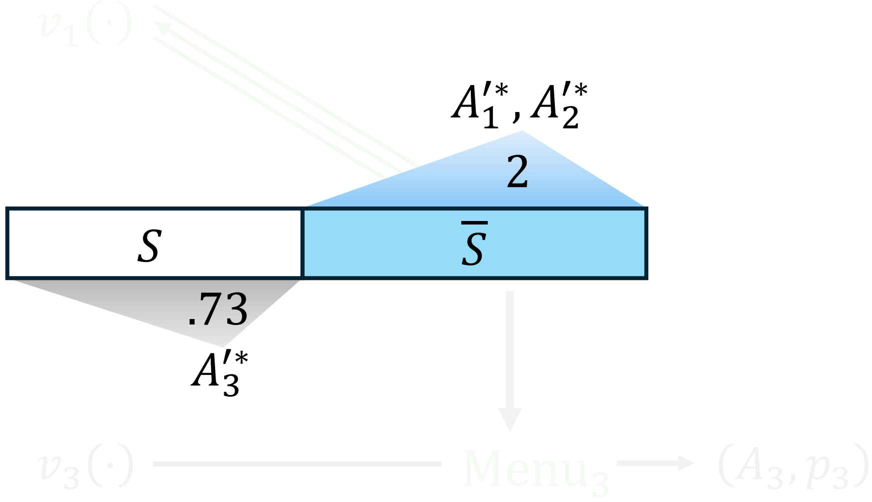
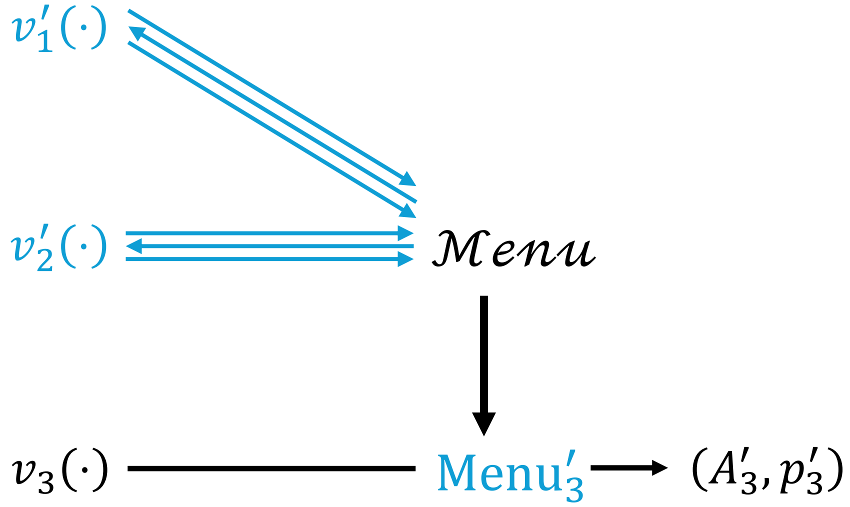
($\frac{1}{2}$-approximation hardness between (v'_1, v'_2))



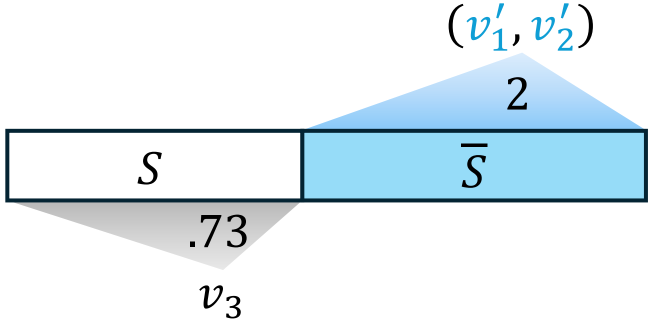
Example: when S is expensive



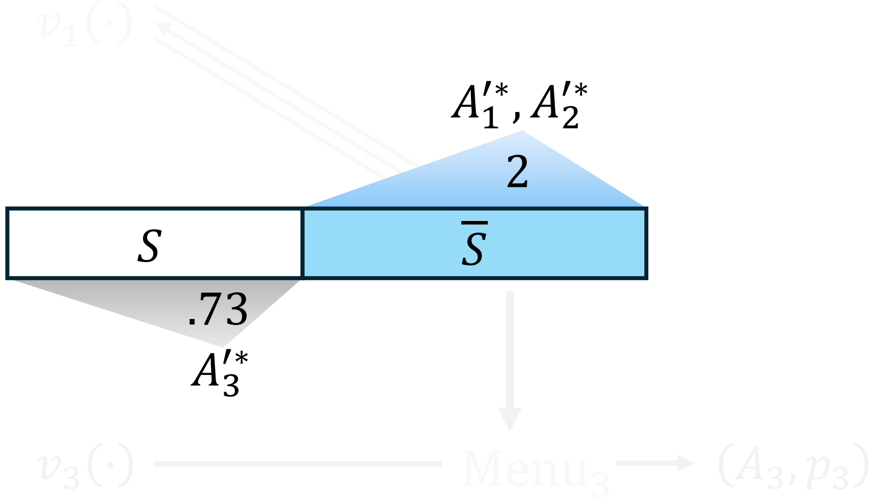


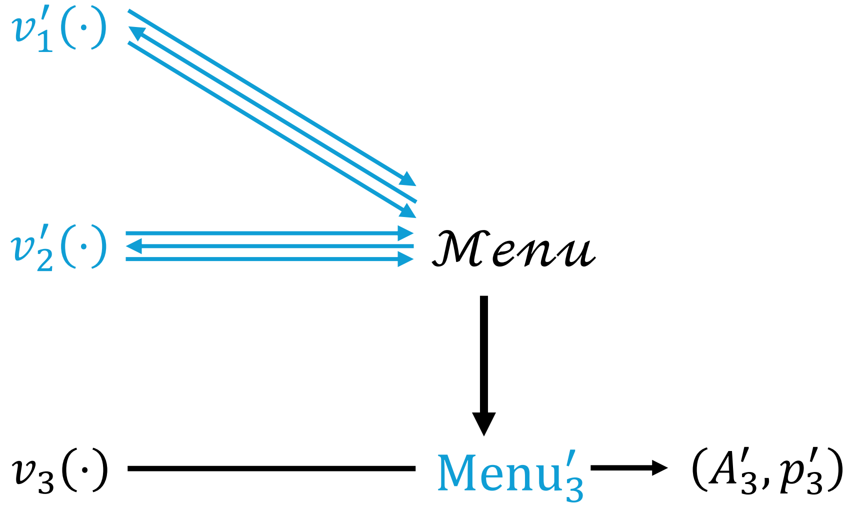
- When $\text{Menu}_3'(S) > 0.73$
  - $A_3' = \emptyset$
  - $A_1' \cup A_2' \subseteq [m]$

  } welfare = **1**
  - However, optimal $A_1'^*, A_2'^*, A_3'^*$ give welfare 2.73



Example: when S is expensive





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  - $A_3' = \emptyset$
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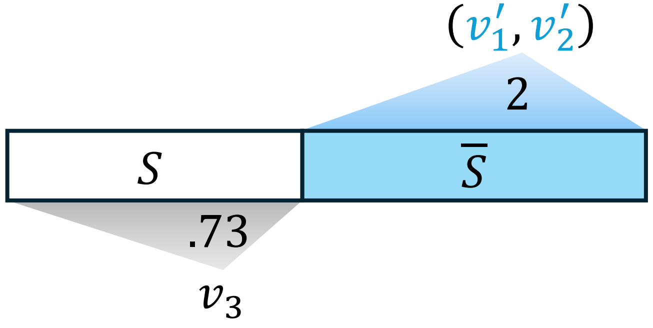
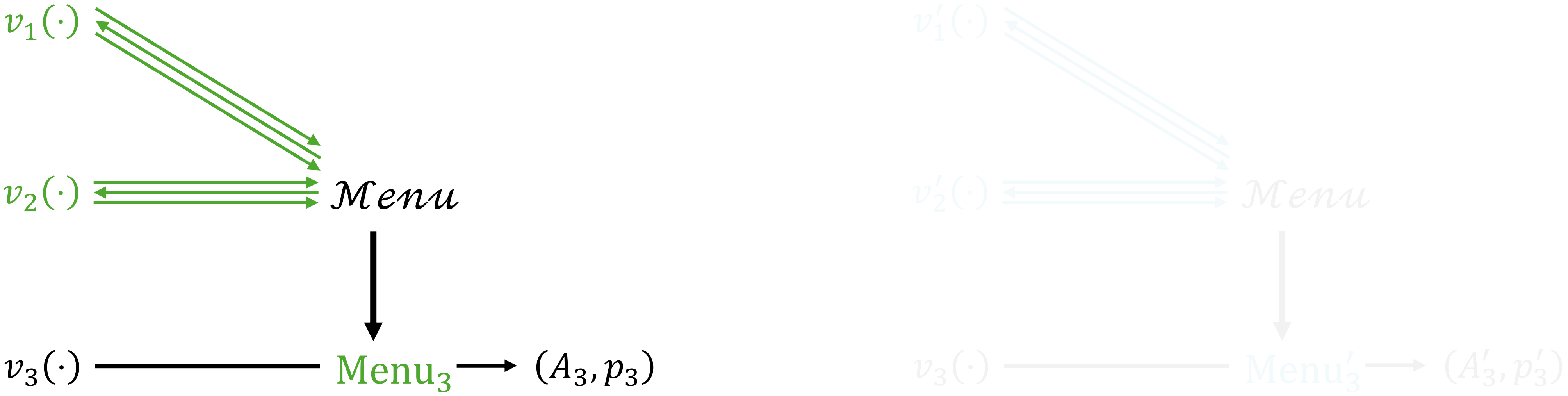
  - 0.366−approximation!

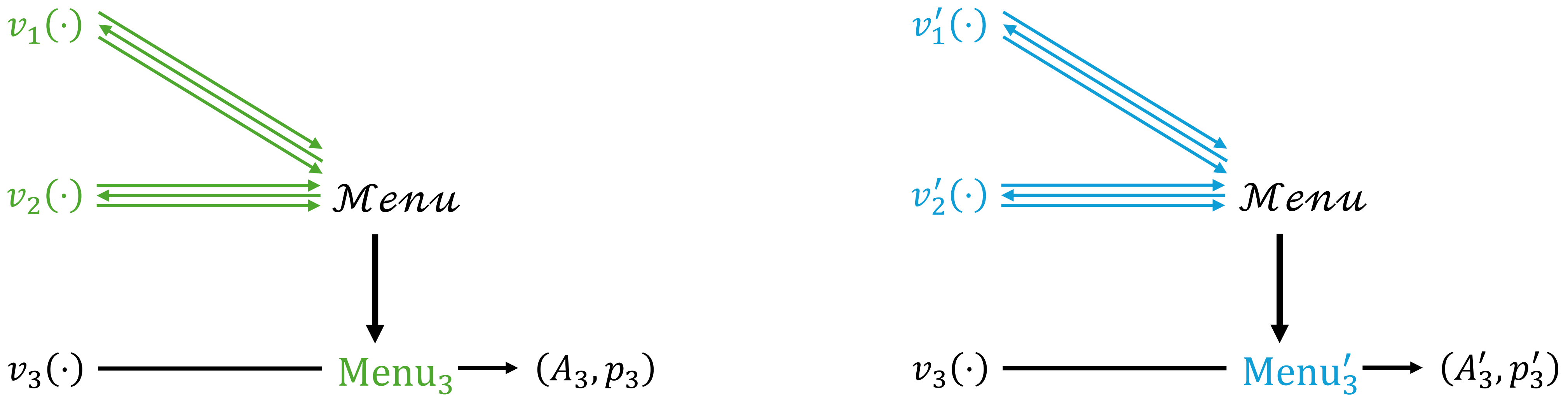


Example: summary



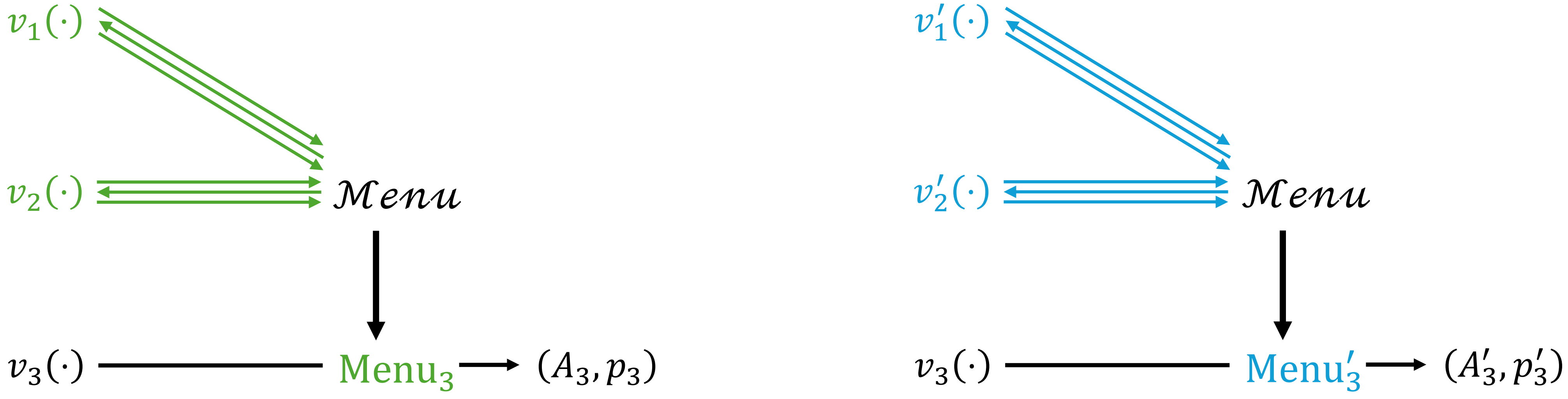
- $\text{Menu}_3(S) \leq 0.73 \;\Rightarrow\;$ 0.366-approximation on (v_1, v_2, v_3)

Example: summary



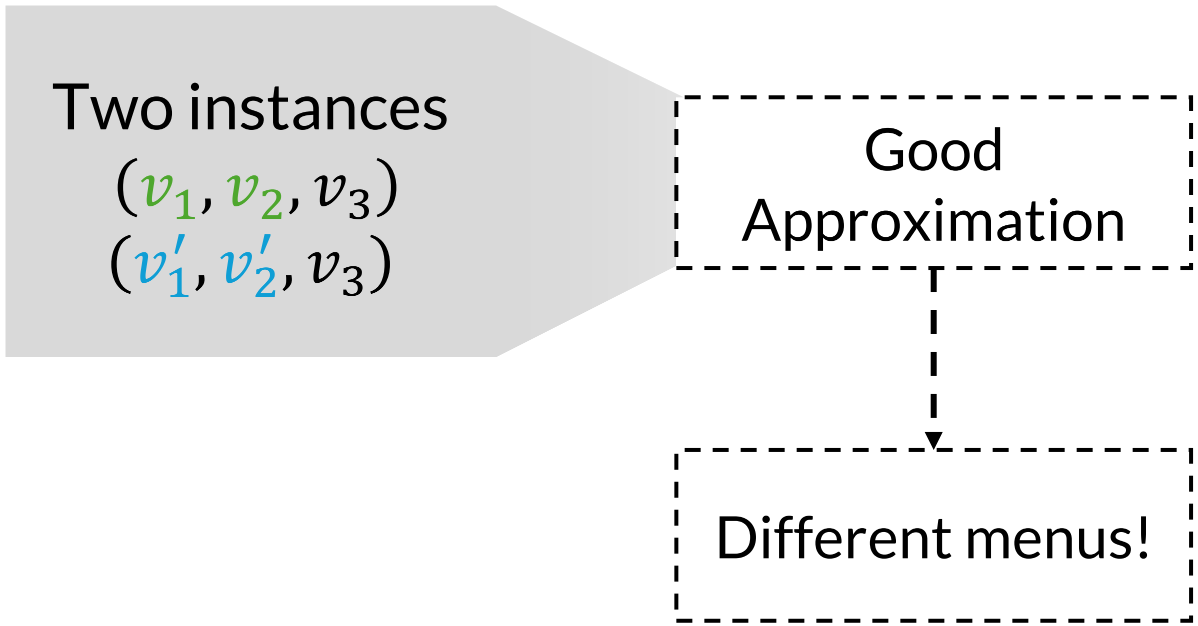
- $\text{Menu}_3(S) \leq 0.73 \;\Rightarrow\;$ 0.366-approximation on (v_1, v_2, v_3)
- $\text{Menu}_3'(S) > 0.73 \;\Rightarrow\;$ 0.366-approximation on (v_1', v_2', v_3)

Example: summary



- $\text{Menu}_3(S) \leq 0.73 \;\Rightarrow\;$ 0.366-approximation on (v_1, v_2, v_3)
- $\text{Menu}_3'(S) > 0.73 \;\Rightarrow\;$ 0.366-approximation on (v_1', v_2', v_3)
- $> \mathbf{0.366}$**-approximation** $\;\Rightarrow\;$ $\mathbf{Menu_3} \neq \mathbf{Menu_3'}$

To implement the idea…



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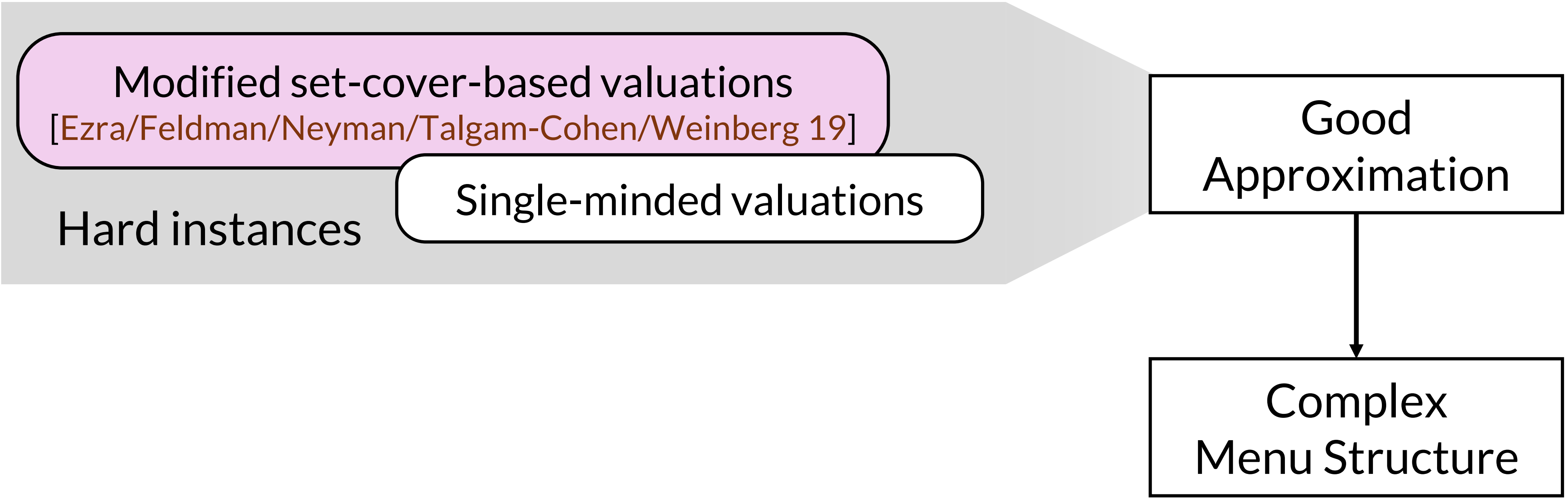

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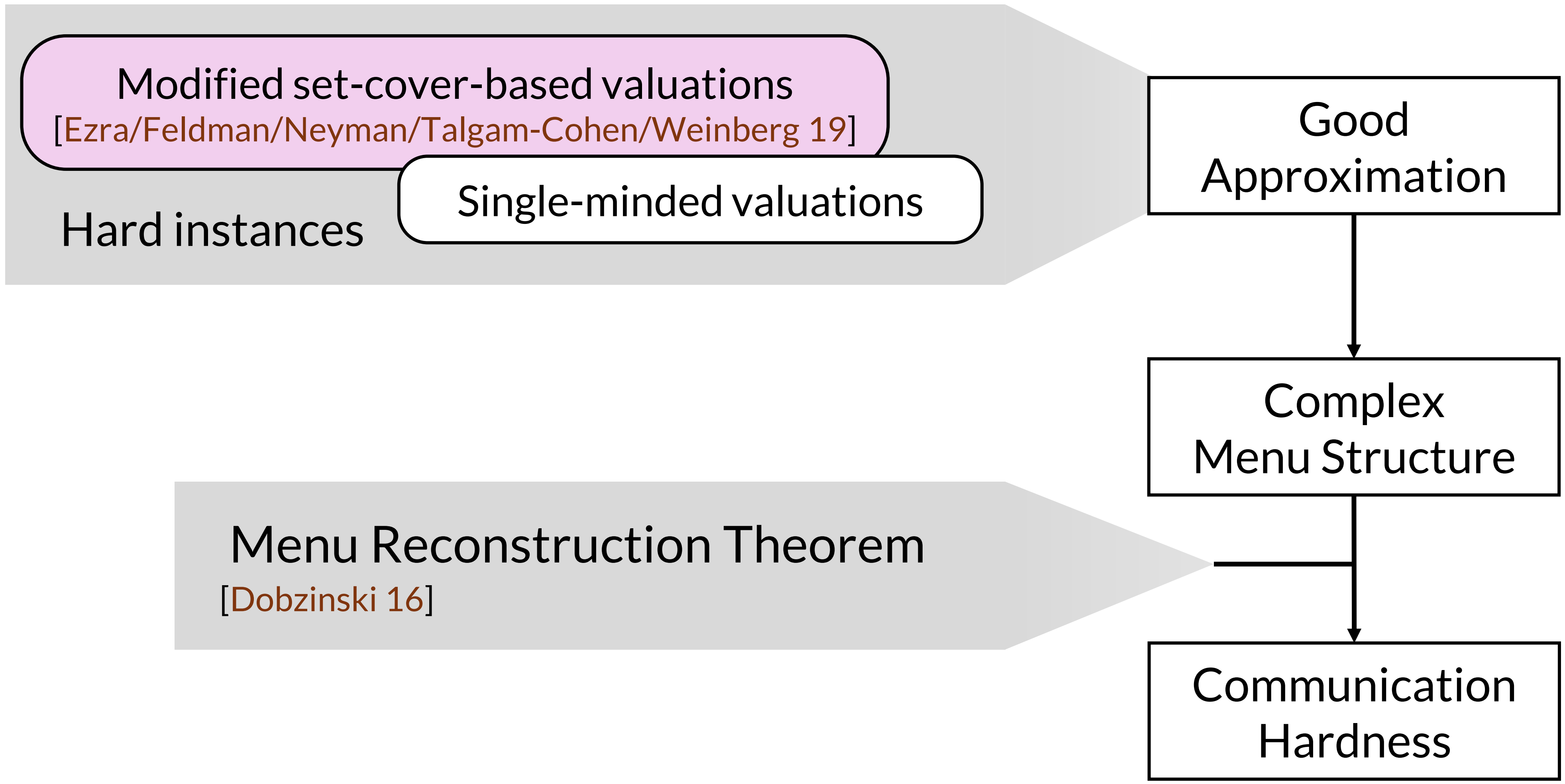

Conclusion

- **Main result**: Separation of truthful and non-truthful protocols

  for 3 $\mathrm{SubAdditive} \cup \mathrm{SingleMinded}$ bidders
  - **Impossibility of 0.366-apx deterministic efficient truthful mechanisms**
  - Existence of 0.5-apx deterministic efficient non-truthful protocols

Conclusion

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  for 3 $\mathrm{SubAdditive} \cup \mathrm{SingleMinded}$ bidders
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Thank you!