Oblivious Online Contention Resolution Schemes

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Single-Item OCRS

- **Input**: integer n, vector $\boldsymbol{x}$, and n elements' statuses (being active or not).
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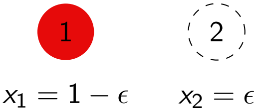

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- A $\frac{1}{4}$-**selectable oblivious** OCRS: Always accept w.p. $\frac{1}{2}$ whenever possible.

History

1. CRS is first formalized by Chekuri, Vondrák, Zenklusen [CVZ14] for rounding fractional solutions in submodular function maximization.
2. OCRS is introduced by Feldman, Svensson, Zenklusen [FSZ16]. It turns out to be a powerful tool for a wide range of applications in Bayesian and stochastic online optimization problems, such as prophet inequalities and stochastic probing.

Our Results

1. We give a simple yet optimal $\frac{1}{e}$-selectable oblivious single-item OCRS.
2. We show that no good CRS or OCRS with $O(1)$ samples exists for graphic or transversal matroids.

A $\frac{1}{e}$-selectable Oblivious Single-Item OCRS

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- **Analysis**: $\frac{1}{e}$-selectable by direct calculation

$$\Pr[i \text{ accepted} \mid i \text{ active}] = \frac{1}{2}\left[\prod_{j<i}(1-x_j) + \sum_{j<i} x_j \prod_{k<i, k\neq j}(1-x_k)\right] \geq \frac{1}{e}.$$

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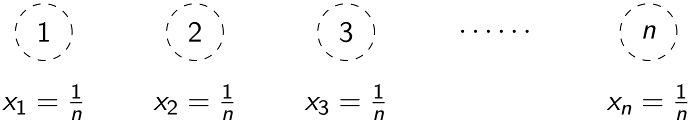
- Minimum is obtained via tha uniform instance $\boldsymbol{x} = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$



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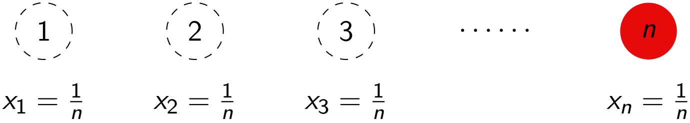

Optimality among Counting-Based Strategies

- A **counting-based strategy** with an infinite sequence of probabilities $(p_1, p_2, \ldots)$:

  When the OCRS sees the k-th active element,
  it accepts (and stops) with probability p_k.

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$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots\right).$$

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$$\left(\frac{1}{2}, 1, 0, 0, \ldots\right).$$

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- The $\frac{1}{e}$-selectable OCRS: a counting-based strategy with probabilities

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- **No counting-based strategy can do better than $\frac{1}{e}$ on uniform instances!**

From Counting-Based Strategies to General Strategies

- Any oblivious OCRS A for size-N input can be characterized by $f_A : 2^{[N]} \to [0,1]$:

  $f_A(T) = \Pr[A$ accepts $(\max T)$-th element $\mid T$ is the set of active elements so far$]$.

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Definition ((ϵ, k)-approximation)

An oblivious OCRS A is (ϵ, k)-approximate to a counting-based strategy C on S if

$$f_A(T) \in [p_{|T|}, p_{|T|} + \epsilon] \qquad \text{for all } T \subseteq S \text{ and } |T| \leq k,$$

where $(p_1, p_2, \cdots)$ is the probability sequence of C.

Optimality in General

Lemma (Any oblivious OCRS is partly counting-based, informal)

For any oblivious OCRS A of size-N input, we can always find a subset $S \subseteq [N]$ and a counting-based strategy C, such that A is $(\epsilon, |S|)$-approximate to C on S.
Moreover, $|S|$ can be arbitrarily large, as long as N is sufficiently large.

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- Having the lemma, one can easily embed the hard instances to these subsets and prove the general $\frac{1}{e}$ lower bound.

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  - Let $\epsilon = \frac{1}{4}$, $N = 5$.

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$f_A(\{i\})$	0.4	0.1	0.7	0.3	1.0

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$f_A(\{i\})$	$[\frac{1}{4}, \frac{1}{2})$	$[0, \frac{1}{4})$	$[\frac{1}{2}, \frac{3}{4})$	$[\frac{1}{4}, \frac{1}{2})$	$[\frac{3}{4}, 1]$

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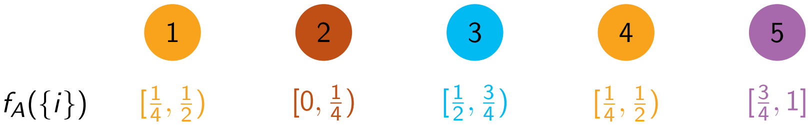

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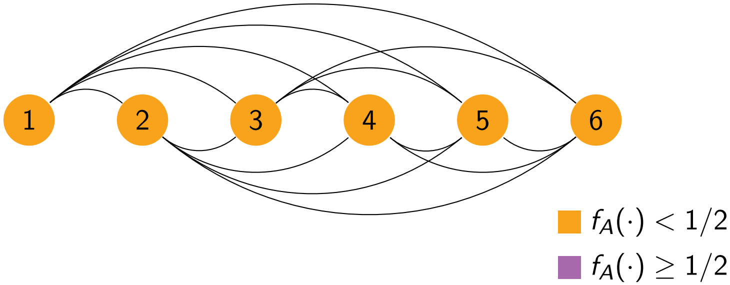

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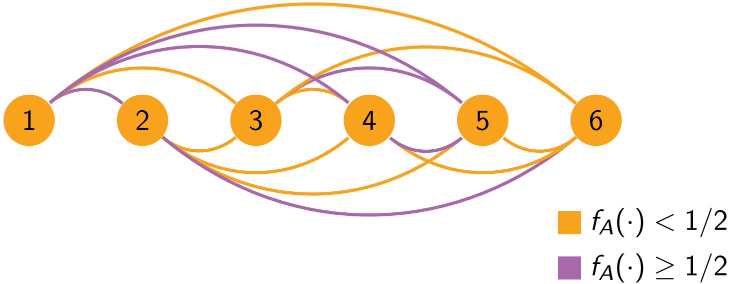

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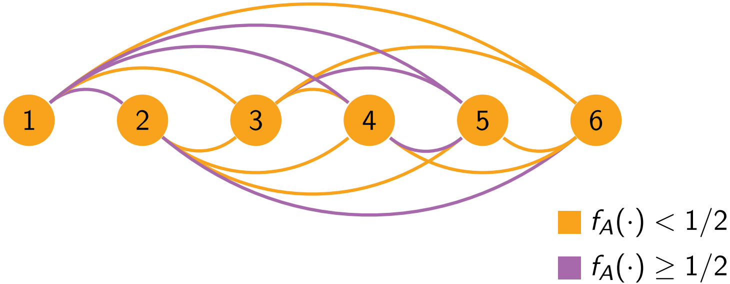
  - Color edge (i, j) with color $\lfloor \frac{1}{\epsilon} f_A(\{i, j\}) \rfloor$.
  - By Ramsey Theorem, such S_2 of size at least 3 exists. $(R(3, 3) = 6)$.



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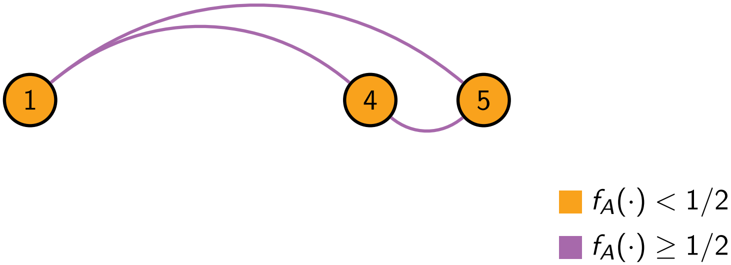

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Hypergraph Ramsey Theorem

For a sufficiently large N (w.r.t. k, c, m), any complete k-uniform hypergraph with more than N vertices and c colors has a monochromatic clique of size m.

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Proof Sketch of the Lemma

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$$\text{Pigeonhole principle} \quad \Rightarrow \quad (\epsilon, 1)\text{-approximation on } S_1 \subseteq [N]$$

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Ramsey Theorem for 3-uniform hypergraph	$\Rightarrow$	$(\epsilon, 3)$-approximation on $S_3 \subseteq S_2$

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Theorem

For any $\epsilon > 0$, no single-item oblivious OCRS is $(\frac{1}{e} + \epsilon)$-selectable.

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For any $\epsilon > 0$, no single-item oblivious OCRS is $(\frac{1}{e} + \epsilon)$-selectable.

1. Show no counting-based strategy can be strictly better than $\frac{1}{e}$-selectable.
2. Prove for any oblivious OCRS, there must be a subset of elements on which it behaves like a counting-based strategy.
3. Embed the hard instance into the subset, hence the hard instance for counting-based strategies applies to all oblivious schemes.

Impossiblity of Oblivious CRS/OCRS for General Matroids

Theorem

For any $c \in (0, 1]$*, there is no oblivious* c*-balanced CRS for graphic matroids or transversal matroids.*
Moreover, the impossibility persists even if the CRS has access to $O(1)$ *samples of the random set* R *of active elements.*

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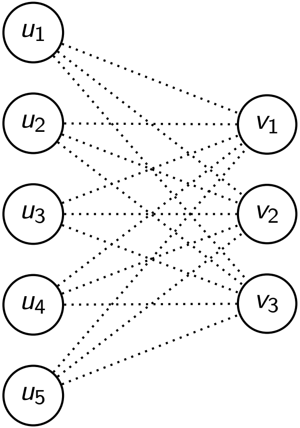
- Idea: hide some always-active elements.



$U = \{\text{edges}\}$

$\mathcal{F} = \{\text{spanning forests}\}$

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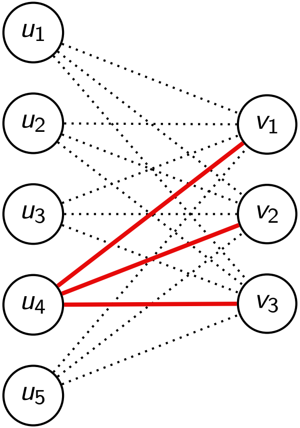
For any $c \in (0,1]$, there is no oblivious c-balanced CRS for graphic matroids or transversal matroids.
Moreover, the impossibility persists even if the CRS has access to $O(1)$ samples of the random set R of active elements.

- Idea: hide some always-active elements.
- E.g., in the graphical matroid $(U, \mathcal{F})$, let $(u_4, v_1), (u_4, v_2), (u_4, v_3)$ be active w.p. 1, while other edges are active with a small probability $\frac{1}{M}$.



$U = \{\text{edges}\}$

$\mathcal{F} = \{\text{spanning forests}\}$

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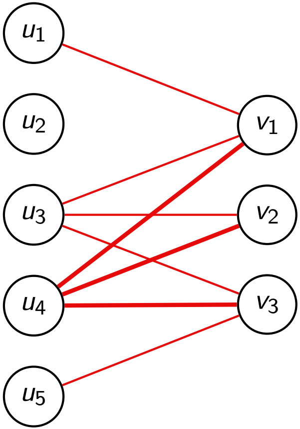

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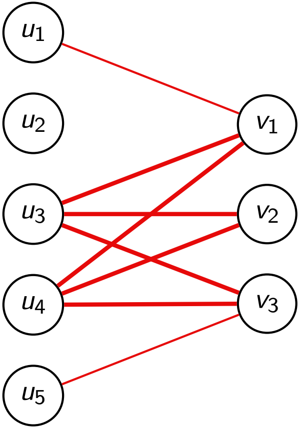
- If $N \gg M^M$, these elements will be instinguishable from others!



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[FSZ16] Moran Feldman, Ola Svensson, and Rico Zenklusen. Online contention resolution schemes. In Robert Krauthgamer, editor, Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2016, Arlington, VA, USA, January 10-12, 2016, pages 1014–1033. SIAM, 2016.